Cognitive Biases

So far we've done:$\rightleftharpoons$

- "quasi-maximization" models that posit that at each moment in time a person is constrained-maximizing a well-defined utility function—just not the 'right' one.$\rightleftharpoons$

Now we'll do:$\rightleftharpoons$

- "quasi-Bayesian" models that posit a person is maximizing utility w.r.t. to beliefs that are in error, as modeled by some specific distortion of Bayesian information processing.$\rightleftharpoons$

What does the 'quasi' mean?$\rightleftharpoons$ Definitions and synonyms:$\rightleftharpoons$

- resembling; seeming; virtual; having some, but not all, of the features of; as if.

- Before outlining errors, an aside ...

An aside on Managed Funds

- Suppose that you observed the following advertisement, in its entirety from a brokerage firm that will advise you on stocks:

"We value you, the client."

- What would you infer from this statement?

Cognitive Biases

- Without answering, let me make up some statistics (my version of empirical work ...) to make it clear.

Suppose 813 funds in U.S. in business at least the last 2 quarters, and

- .... 202 of them have ads saying
  - "We've beat the market average the last 2 quarters"
- .... 197 of them have ads saying
  - "We beat the market average the last quarter"
- .... 414 of them have ads saying
  - "We value you, the customer"

Now what do you think "We value you, the customer" means?

- It probably means that the mutual fund lost to the market last quarter.

Cognitive Biases

Was my question unfair?$\rightleftharpoons$

- Maybe, maybe not.$\rightleftharpoons$
- I didn't give you context/comparison.$\rightleftharpoons$
- But not at all clear that the mutual fund would give you context.$\rightleftharpoons$
- Without context, you don't know what to make of it. Maybe salient if you see lots of the other ads; depends.$\rightleftharpoons$
- Those that have something good to report, report.$\rightleftharpoons$
- Those who don't, tell you how valuable you are to them.

$\looparrowright$

Cognitive Biases

Issues of what you pay attention to, including the strategic logic of situations, the role of "dogs not barking", etc., may play a huge role in inference.

- Not just whether you use appropriate Bayesian updating or appropriate strategic logic given the information you focus on
- But whether you focus on all the questions you should focus on.
- We'll get at a bit at end.

- But this wasn't really the topic I wanted to pursue $\rightleftharpoons$

Suppose that all 414 of them saying they value you lost to the market average last quarter. $\rightleftharpoons$

- An easy question to segue into next topic: $\rightleftharpoons$

What obvious inference would you make from the three different ads. $\rightleftharpoons$

- What is the quality of a fund that advertises it has beat the market two quarters in a row vs. the other two categories?

$\looparrowright$

Cognitive Biases

Probably ... they are of the same quality. They are all probably average.

- Why?

Very close to the distribution you'd get if they were all average.

- Randomly 25% would beat market twice in a row, 50% would lose the last quarter.

I haven't given much to go on, and probably there is a clever way to infer difference in quality.

- But I'd argue these statistics are suggestive that all funds are average.

Cognitive Biases

What if you did not easily know the distribution of performances?

- What should you infer from seeing good recent performance?
- Should you pay to get good evidence on recent performance?

We happen to know such distributions are true.

- But what if investors don't?

It could be argued (and it would be correct) that they *should* know

- Logic of financial markets should tell them not to search for patterns

Final topics of course related to that.

- Will turn out (among other lessons):
    - Often see patterns where there are none (cognitive biases),
    - and where you should know a priori that there are none! (cursedness)

Enough of me doing finance.

Cognitive Biases

Outline:

1. Modeling Cognitive Errors
2. LSN/NBLLN/SSN in sample prediction
3. Bin Effects
4. Disentangling NBLLN and Bin Effects
5. LSN/NBLLN/SSN in inference
6. Base-Rate Neglect
7. Things it would be useful to have more data on

$\looparrowright$

Cognitive Biases

Today's talk based on:

- Two were-surely-going-to-be-finished-by-now unfinished papers
  - "Base-Rate Neglect" with Dan Benjamin and Aaron Bodoh-Creed
  - "Misconceptions of Chance: Evidence from an Integrated Experiment," with Dan Benjamin and Don Moore (previous draft on line)
  - "Belief Movement, Uncertainty Reduction, and Rational Updating," with Ned Augenblick
- Some existing papers of mine:
  - "Inference By Believers in the Law of Small Numbers"
  - "The Gambler's and Hot-Hand Fallacies" with Dimitri Vayanos
  - "A Model of Non-Belief in the Law of Large Numbers," with Dan Benjamin and Collin Raymond
- And mostly:
  - Work above and today based on lots and lots of existing research ... **Griffin and Tversky**, Tversky and Koehler, gobs of Kahneman, Tversky, others.

Cognitive Biases

Approach building from research under the broad heading of “judgment and decisionmaking" (JDM)⇌

- How people’s probabilistic judgments might be distorted.⇌
- Probabilistic reasoning *not* random or totally irrational.⇌
- Human rationality, not superhuman rationality or subhuman idiocy.⇌

Types of formal bias models:⇌ “Misfunctional Bayesian”: ⇌

- Use conditionals and priors, but with wrong functional form.⇌
  - Base-rate neglect.⇌

But many models of errors stick closer:⇌ Quasi–Bayesian Models⇌

- Assume people engage in putatively proper Bayesian updating.⇌
- But specify a precise way in which they either mis-observe or mis-understand how that evidence relates to the hypotheses.⇌
- Examine the implications of Bayesian updating given the error.

- Two categories of quasi-Bayesian

Warped-Model Bayesian:

- False (but internally consistent) model of how signals are generated.
    - Barberis-Shleifer-Vishny (1998),
    - Rabin (2002), Rabin and Vayanos (2010),

Information-Misreading Bayesian:

- Right model, but misread signals.
    - Rabin and Schrag (1999)
    - Mullainathan (2002).

Cognitive Biases

Different combinations of bias and environment can lead to:

- Overinference/overconfidence ... infer "too much" from information.
- Underinference/underconfidence ... people infer too little

These are **manifestations** of biases, not types of biases.

- Psychologists first, now economists: talk like direction of beliefs vs. appropriate Bayesian are general tendencies.
- Now too many researchers pitch generic "Overconfidence"/overinference
- There is no general tendency towards over- or under-inference
  - And no great definition outside of binary questions

Sampling Biases$_{\rightleftharpoons}$

Now review just subset of biases$_{\rightleftharpoons}$

- **The** (!) important biases in the simplest setting:$_{\rightleftharpoons}$
- Take an *i.i.d.* process *should* know is *i.i.d.*, or at some level *do* know.$_{\rightleftharpoons}$
- Prototype: (possibly biased) coin yields h in proportion $\theta \in (0, 1)$.$_{\rightleftharpoons}$
  - What are beliefs about samples given θ?$_{\rightleftharpoons}$
  - What do people infer about θ from samples?

$\looparrowright$

Cognitive Biases

In coins & urns, essentially 3 biases:

- Law of Small Numbers and Gambler's Fallacy:
  - Tversky and Kahneman (1971): exaggerated belief that small samples and short streaks will reflect population mean.
- Non-belief in LLN:
  - Very large psychology literature (1965ish-1975ish) on "conservatism."
  - BRR (2012) meta-analysis: under-inference from non-small samples.
- Base-Rate Neglect
  - Underweighting priors when processing new information

Cognitive Biases

LSN, GF, and NBLLN are, as primitives, distortions in beliefs about likelihood of different samples being generated from given θ.

- Bias: instead of Bayesian $p(s|\theta)$, some $\widetilde{p}(s|\theta)$.
  - Rabin (2002), RV (2010), & BRR (2012)
- These theories predict misinference from assuming

$$p(\theta|s) = \frac{\widetilde{p}(s|\theta)p(\theta)}{\sum_{\theta'} \widetilde{p}(s|\theta')p(\theta')}.$$

- Ignore BRN for now
- Seems like is close to true:
  - *Except* for BRN, inference errors more or less accord with Bayesian inference applied to sample-prediction errors.

Now: evidence on ways $\widetilde{p}(s|\theta) \neq p(s|\theta)$.

- GF/LSN, NBLLN $\longrightarrow$ SSN
- We'll come back to inference problems.

Cognitive Biases

Gambler's Fallacy

- Evidence in BMR (2013, 2016)
- But better evidence from earlier:
- Maryland State Pick-Three Lottery
  - Pari-mutuel Betting provides good data: Infer numbers from odds created, AND have people losing money from bad beliefs (many prediction tasks i.i.d. world means no wrong behavior).
  - Bet 50 cents on day's 3-digit draw, and winners get 52% of total bets (this is typical state cut).
  - If $\frac{1}{10}\%$ people bet on a number, it pays \$260. More than \$260 means $< \frac{1}{10}\%$ bet on it; less than \$260 means $> \frac{1}{10}\%$.

Cognitive Biases

Terrel (1994) reported average winnings as function of when the last time that number won

Within Week:	*\$349*
1-2 Weeks ago:	*\$349*
2-3 Weeks ago:	*\$308*
3-8 Weeks ago:	*\$301*
Not Within 8 Weeks:	*\$260*
Overall:	*\$262*

E.g., 25% fewer bet on number if won in last 2 weeks. Expected return 34% higher betting on recent winners than recent losers.

What do people predict about samples?

KT (1973): How likely different proportions heads in a 50/50 coin?

		45-55%	75-85%
$N = 10$	true	25%	4%
	people think	20%	6%
$N = 100$	true	68%	$\approx 0\%$
	people think	22%	5%
$N = 1000$	true	$\approx 100\%$	$\approx 0\%$
	people think	21%	5%

Cognitive Biases

- Already by N = 10, distribution too dispersed.$\rightleftharpoons$
  - By N = 1000, it is extreme.$\rightleftharpoons$
- Now: BMR (2013,2016) replicate, get very similar results, but incentivized and frequentist.$\rightleftharpoons$
- But we have design feature that changes interpretation of some$\rightleftharpoons$
- Also, evidence for GF won't discuss today$\rightleftharpoons$
  - (After 9 heads, people expect $\frac{2}{3}$ chance tails.)$\rightleftharpoons$

Posted 2013 version, but now report updated data too. $\rightleftharpoons$

- But first, aside central to our experiment

$\looparrowright$

Support Theory and Bin Effects

- (Close cousin to stuff probably very familiar to you)
- Per Tversky and Koehler (1994) and other models, small probability events exaggerated, and combining events reduces total weight.
- Suppose elicit beliefs p on exhaustive/exclusive 3 events $\{A, B, C\}$,
  - $p(A) + p(B) + p(C) = 1.$
- And elicit q on 2 events $\{A \cup B, C\}$,
  - $q(A \cup B) + q(C) = 1.$
- Then will find: $q(A \cup B) < p(A) + p(B)$ and $q(C) > p(C)$.
  - **Important corollary: tendency to overestimate small probabilities**

Cognitive Biases

BMR experiment:

- Skipping details on design.
  - Posted and future paper has details
- 100 subjects Pittsburgh food court, 300 subjects Berkeley experimental lab

Motivation for Experiment:

- Incentivized evidence relatively sparse. Especially for NBLLN.
- Eliminate confounds one might sensibly worry about.
- Eliminate confounds one might non-sensibly worry about.
- Triangulation
  - **Within-subject inconsistencies on same data ... even exotic beliefs about experimental design can't explain.**

Cognitive Biases

- In shared introduction to most questions, we told subjects:
  - "We flipped a coin ten times. Actually, we had a computer simulate the coin flipping, generating exactly the same type of random series real coins do. This was a fair coin, in the sense that coin could come up either heads or tails, and there was an equal chance of each. That generated one ten-flip set. Then we did it again, and again, and again, until...we had 1 million ten-flip sets."
- Generated 1 million samples each size using the Matlab randomizer.
- **All questions elicit beliefs regarding (single) generated sample.**
- Entirely within-subject design

Histograms

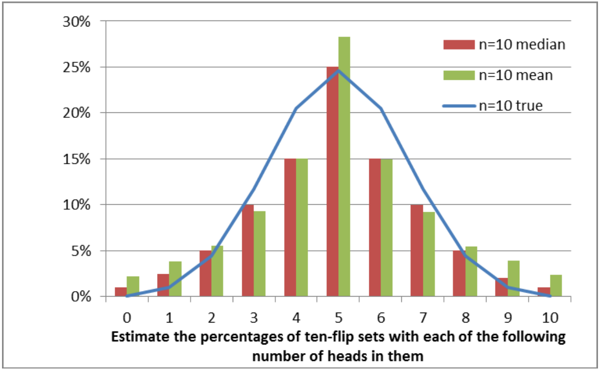
- Eliciting beliefs about the frequency distribution of outcomes.
- Subjects typed in a number between 0 and 100 for each group.
- Screen showed the sum of the percentages.
- Required sum = 100% before could continue to next screen.

Cognitive Biases

- NBLLN?⇌ ... (Once controlled for bin effects?)⇌
    - Basic answer: Yes, for 1,000 and 1,000,000⇌
    - **Not** too dispersed for $N = 10$, once bin effects controlled for⇌
    - Confirm KT SSN claim: no differences 10, 100, 1k, 1m⇌
    - But shockingly little rebellion against CLT in 1,000,000 case! ... ⇌
        - Non-belief in LLN rather than disbelief in LLN?
- Clear and Big "Bin Effects"⇌
    - Beliefs 5/10 heads from 20% to 36% by binning more coarsely;⇌
    - Beliefs on 451-549 / 1000 heads from 19% to 40%.⇌
- Were indeed confounding results⇌
- Smoking-gun approach:⇌ **Within partition:** ⇌
    - when right answer $\pi(A) \leq \pi(B)$, subjects believe $p(A) > p(B)$⇌
    - then bias towards A.⇌
    - (Dan Benjamin and I working on more general exploration of method).

Cognitive Biases

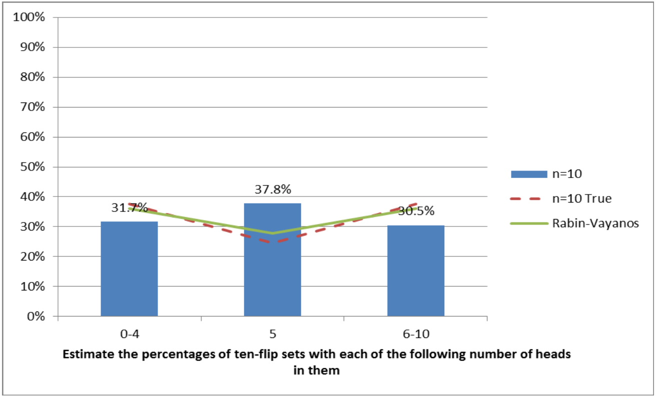
We get "exact representativeness", but else too spread:



- Just like KT earlier results.
- But the "too spread" might just be probability compression

Cognitive Biases

But re-binning reverses the results:$\rightleftharpoons$



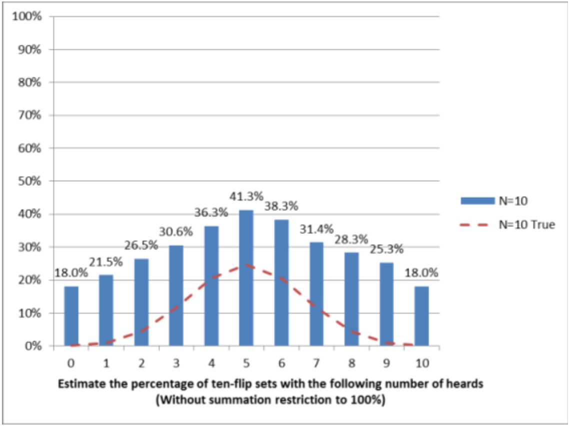
$\rightleftharpoons$

- Note the smoking-gun evidence: $\rightleftharpoons$
  - We know tails too thin because distortion relative to truth opposite of compression

Cognitive Biases

Proof that compression is a confound:

- When done separately, all less-than-50% bins get exaggerated.
- "What is probability of N heads?"



Cognitive Biases

So N=10 over-dispersion bad interpretation. $\rightleftharpoons$

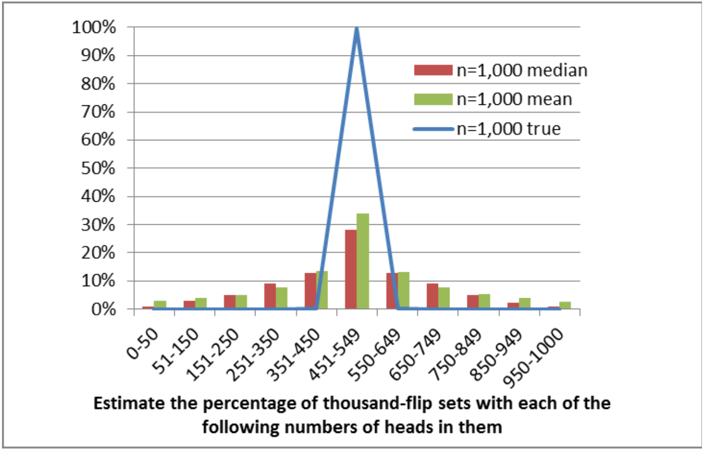
- But what about large samples? $\rightleftharpoons$

NBLLN (sample sizes 1,000 and 1,000,000) $\rightleftharpoons$

- With same eleven-outcome, confounded-with-compression approach, over-dispersion for N=1,000:

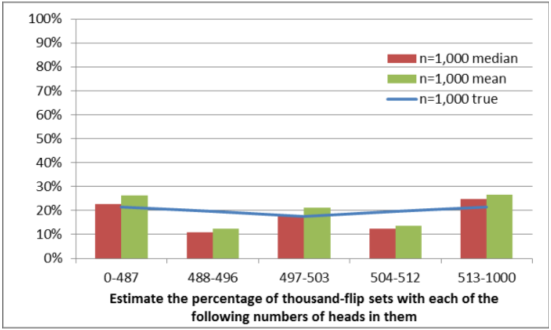
$\looparrowright$

Cognitive Biases



But we can show **not** just binning:⇌

- 5-equal-bins treatment nails it⇌



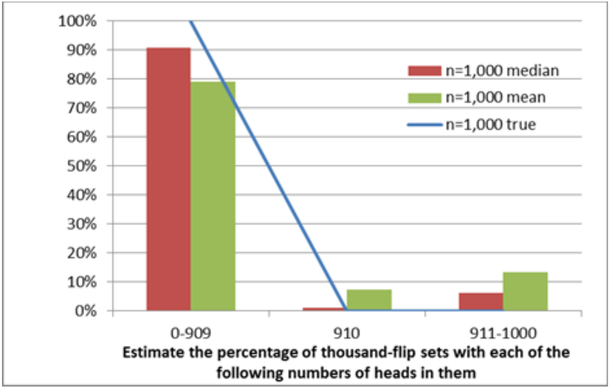
⇌

- "W" may be real representativeness.⇌
- But clear outer tail vs. inner tail result.⇌

My favorite example shows real under-appreciation of CLT:

Cognitive Biases

Probabilities {0-909,910,911-1000}?⇌ Mean {79%,7%,14%}, Median {91%,1%,6%}



⇌

- People don't realize that 911-1,000 never happens.

↬

Cognitive Biases

But do even *we* appreciated thinness of tails?$_{\rightleftharpoons}$

- True fact: $p(910) > 5 \cdot p(911 - 1000)$.$_{\rightleftharpoons}$ (Who knew?)$_{\rightleftharpoons}$
- (Question to contemplate: is the bell curve as optical illusion?)$_{\rightleftharpoons}$

How is our intuition on CLT/LLN?$_{\rightleftharpoons}$

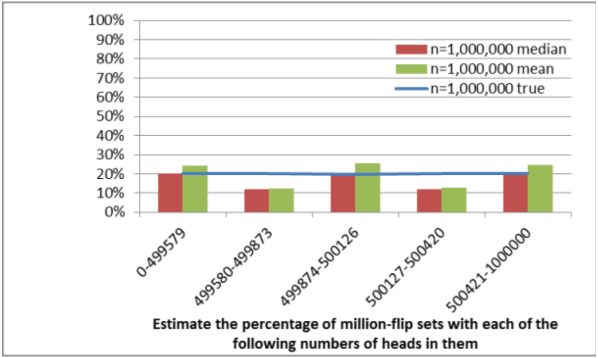
- Probability that
  - Exactly 60% of 40 flips heads? $_{\rightleftharpoons}$
  - Exactly 90% of 40 flips?$_{\rightleftharpoons}$
  - Both very small ...$_{\rightleftharpoons}$ but how small? How compare?$_{\rightleftharpoons}$
- Betting your intuition is bad / off. What is $\frac{prob(36/40)}{prob(24/40)}$? $\rightleftharpoons$
  - About $\frac{1}{680{,}000}$. $\rightleftharpoons$
  - Not knowing exact $\longrightarrow$ "bound error";$_{\rightleftharpoons}$
  - finding this shocking $\longrightarrow$ "astray error".

$\hookrightarrow$

Cognitive Biases

Also elicited beliefs about sample size 1,000,000

- First time ever such beliefs have been elicited?
- DB and MR astonished by results. **"Nuance"** not reporting today
- Non-belief in LLN we think is right, but vigorous disbelief less true than we thought?



Evidence on LSN, NBLLN, & SSN in Inference

Beautiful experiment: Griffin and Tversky (1992).

- "Imagine that you are spinning a coin, and recording how often the coin lands heads and how often the coin lands tails. Unlike tossing, which (on average) yields an equal number of heads and tails, spinning a coin leads to a bias favoring one side or the other because of slight imperfections on the rim of the coin (and an uneven distribution of mass). Now imagine that you know that this bias is 3/5. It tends to land on one side 3 out of 5 times. But you do not know if this bias is in favor of heads or in favor of tails."

Cognitive Biases

Tempting model to capture both: Sample-Size Neglect.

- Kahneman & Tversky (1973) and Griffin and Tversky (1992):
  - People attend to proportions, not sample size.
  - Intuitive model, **not far off**.

Cognitive Biases

Two possible $i.i.d.$ coins each with prob $= .5$: $\rightleftharpoons$

- $\pi(h|\cdot) = \frac{3}{5}$ coin and a $\pi(h|\cdot) = \frac{2}{5}$ coin$\rightleftharpoons$
- Observe a set of flips h, t.$\rightleftharpoons$
- Bayes' Law says $\frac{\pi(\theta|h,t)}{\pi(1-\theta|h,t)} \equiv l(h,t) = \left(\frac{3}{2}\right)^{h-t}$.$\rightleftharpoons$
- So Bayesian inference from (h,t) depends solely on $h-t$.$\rightleftharpoons$
- That's all well and good ... but:$\rightleftharpoons$
  - Only geeks think that way.$\rightleftharpoons$
  - And only when paying attention.$\rightleftharpoons$
  - People in fact base beliefs on how close $\frac{h}{h+t}$ looks to $\frac{3}{5}$ vs. $\frac{2}{5}$.$\rightleftharpoons$

A tale of two tables **presenting the same data**:

$\looparrowright$

Over-infer from small samples, under-infer from large samples.

Sample of (h,t)	$h+t$	$h-t$	Median $P(\theta = \frac{3}{5} \mid h, t)$	Proper $B(\theta = \frac{3}{5} \mid$
5,0	5	5	.92	.88
7,2	9	5	.77	.88
11,6	17	5	.64	.88
19,14	33	5	.60	.88
3,0	3	3	.85	.77
4,1	5	3	.80	.77
6,3	9	3	.67	.77
10,7	17	3	.60	.77
2,1	3	1	.63	.60
3,2	5	1	.60	.60
5,4	9	1	.55	.60
9,8	17	1	.54	.60

Cognitive Biases

Meta-Lesson: **proportional thinking**$_{\rightleftharpoons}$

Sample of (h,t)	% heads	Median $P(\theta = \frac{3}{5} \mid h, t)$	Proper $B(\theta = \frac{3}{5} \mid h, t)$
5,0	100%	.92	.88
3,0	100%	.85	.77
4,1	80%	.80	.77
7,2	78%	.77	.88
6,3	67%	.67	.77
2,1	67%	.63	.60
11,6	65%	.60	.88
3,2	**60%**	**.60**	**.60**
10,7	**59%**	**.60**	**.77**
19,14	**58%**	**.60**	**.88**
5,4	55%	.55	.60
9,8	53%	.54	.60

$\looparrowright$

Some economic implications?

Implications of LSN? $\rightleftharpoons$

- Rabin (2002) models: false belief by investors in the value of advice/managed funds may be LSN-related. $\rightleftharpoons$
- People over-infer from short-run performance of mutual fund that its manager must be a genius. $\rightleftharpoons$
- "Fictitious Variation": $\rightleftharpoons$
  - When looking at global data, in fact might infer skill out there in investing when there is none. $\rightleftharpoons$
- Rabin and Vayanos (2010) also explain other investment errors, such as under- and over-reaction and false belief in hot hands. $\rightleftharpoons$
- More generally: How and when GF$\longrightarrow$ HH.

$\looparrowright$

Implications of NBLLN?

- People unconvinced by statistics.
- And: NBLLN is underemphasized as a *necessary* enabler and almost-sure confound for "over-extraction" biases.
  - NBLLN says under-infer from Consumer Reports data sets.
  - Salience/vividness: infer too much from friend's bad (& costly) experience.
  - But: even if over-weighted your friend by 500 times, still follow *Consumer Reports* if believe in LLN.
  - Even the greats have been sort of mis-emphasizing their own results.
- Exacerbates risk/loss aversion:
  - People exaggerate chance of losing from large number of better-than-fair bets.
  - Not just about narrow bracketing.

Implications of Bin Effects? $\rightleftharpoons$

- Is it just an elicitation confound? $\rightleftharpoons$
  - To ID nature of bias, we "control" for it. $\rightleftharpoons$
  - But not clear that there is any such thing as "true" preference, independent from bin effects. $\rightleftharpoons$
  - And, presumably, also a real thing economically. $\rightleftharpoons$
- If people act as if the total probability of cancer is higher when broken down by different types, then ... $\rightleftharpoons$
  - Means structure of doctor communication, insurance, etc., matter.

$\looparrowright$

Cognitive Biases

Now: Base-Rate Neglect $\rightleftharpoons$

- Model (stated, estimated over the years): $\rightleftharpoons$
- Base-rate-neglect sufferer (**Saki**) believes:

$$p(\theta|s) = \frac{p(s|\theta)p(\theta)^{\alpha}}{\sum_{\theta' \in \Theta} p(s|\theta')p(\theta')^{\alpha}}. \rightleftharpoons$$

- where $\alpha \in [0, 1)$. $\rightleftharpoons$ Extreme $\alpha = 0$ Saki:

$$p(\theta|s) = \frac{p(s|\theta)}{\sum_{\theta' \in \Theta} p(s|\theta')}. \rightleftharpoons$$

- Combine LSN, GF, and NBLLN with BRN: $\rightleftharpoons$

$$p(\theta|s) = \frac{\widetilde{p}(s|\theta)p(\theta)^{\alpha}}{\sum_{\theta'} \widetilde{p}(s|\theta')p(\theta')^{\alpha}}.$$

Cognitive Biases

The textbook example of BRN:$\rightleftharpoons$

- Test for disease is 90% accurate (symmetrically)$\rightleftharpoons$
- 5% of tested population have disease$\rightleftharpoons$

If test positive, probability of disease?$\rightleftharpoons$

- Stats text: tells you right answer (32%)$\rightleftharpoons$
- Psych articles: shows people give wrong answer (say, 90%).$\rightleftharpoons$

But ... $\rightleftharpoons$

- You might be left with wrong impression from BRN ...

$\looparrowright$

Cognitive Biases

Test	Priors	Posteriors		Frequency
		Tommy	Saki	
Positive	5%	32%	90%	14%
Negative	5%	$< 1\%$	10%	86%

- Yes, her beliefs have moved too much
  - Tommy beliefs moved (up) 27% or (down) 5%.
  - Saki's beliefs moved (up) 85% or (up) 5%.
- But, while Saki beliefs too extreme after positive result ...
  - (which is the main image of BRN)
  - But too **moderate** after negative result
- Turns out a general feature of BRN:
  - Beliefs move too much, but are too moderate
  - In dynamic, i.i.d. model, ergodic non-convergence.

Cognitive Biases

- In fact, a bizarre “moderation effect”:
  - buried in definition, buried in literature

Test	Priors	Posteriors		Frequency
		Tommy	Saki	
Positive	5%	32%	90%	14%
Negative	**5%**	$< 1\%$	**10%**	86%

- After **negative** signal, her beliefs move **up.**
- Wouldn’t necessarily bet on that implication here.
  - But it is an implication that the model must own ...
  - And just maybe wants to own ...

Cognitive Biases

Griffin and Tversky (1992) also best evidence on BRN?

- One of very few examples where signal, base rates same direction
- And, they find the extreme moderation effect ...
  - Subjects' with priors 90% chance heads biased, given sample of $(6h, 4t)$, posteriors $< 90\%$.

Cognitive Biases

Fleshing out model in dynamic contexts.

- **When signals arrive sequentially**, cumulative updating?
  - Combine all signals together, then combine with original base rate?
  - Or sequentially update—each new signal generates new base rate?
- We assume 2nd—central to many results.
  - so today's posteriors are tomorrow's priors.
- Data observed in the recent past matters more than data from distant past (even though signals i.i.d.).
  - So, obviously, beliefs from same information depends on order.
  - Forever and ever.
  - As if current day's temperature affects belief in global warming?
    - (Yi, Johnson, and Zaval, 2011).

Summary of implications:

- Recency effect: more recent data matters more.
- Long-run beliefs ergodic
  - initial prior ceases to matter.
  - *Support* of ergodic distribution independent of true state.
  - Never become fully confident.
- Moderation effect: strong priors always dampened.

Things it would be useful to have data on.

- We should gather direct evidence on beliefs across domains.
    - Is happening some
    - Should happen more
    - **Will** happen more
- For identifying biases, we need histograms.
    - So please help
- You can help us understand how widespread lab errors are in world
    - Evidence is on coins & urns, but want evidence on realer things.
- People's beliefs about how others would interpret given information.
    - Directly gets at issues of interpersonal thinking.
    - But also indirectly at nature of biases, especially confirmatory.
- Prospective beliefs
    - Huge: beliefs about future updating
    - Virtually unmentioned in the psychology
    - We've been making models without any evidence

Cognitive Biases

Problems and Issues in Non-Bayesian Models

- Are these problems, or are they "features"?
  - Both
- We've seen some of the inconsistencies:
- BMR (2014) show:
  - NBLLN does not come from **any** theory of sequences.
  - Will give logically inconsistent answers for different questions.
- Not just wrong probabilities. But mutually inconsistent.
- BSV, R, RV, RS, M are quasi-Bayesian.
  - People make errors
  - But internally consistent.
  - (if don't look too carefully)
- Non-quasi-Bayesian Non-Bayesian models
  - Fundamental "problems".
  - These "problems" *are* problems.
  - But they're also (often) important truths.

Cognitive Biases

NBLLN inherently implies "clumping" issues...

- Suppose Barney reads a summary of 10,000 individuals' experiences in *Consumer Reports*. Then he asks a sequence of 5 friends about their experience driving a Volvo.
- Inherent to NBLLN, it matters whether Barney processes these 10,005 signals as...
- a single sample of size 10,005?
  - BRR call such a Barney "pooling"
- as one group 10,000 signals and five groups of 1 signal each.
  - BRR call such a Barney "acceptive"
- as 10,005 separate signals
  - BRR call such a "Barney" "atomizing".

Cognitive Biases

- Back to NBLLN, where BRR allow 9-fold possibilities on clumping:
  - *Retrospective grouping*: processing of clumps already received for purpose of inference.
  - *Prospective grouping*: forecasted processing of future clumps for subjective sampling distribution or anticipating future inferences.

Cognitive Biases

-

	r-atomizing	**r-acceptive**	**r-pooling**
p-atomizing	*		
p-acceptive		*	#
p-pooling			*

- Diagonals (asterisks) are *processing-consistent*.
  - And Northwest is Tommy.
- BRR discusses case for each of the 9 combinations,
  - but dismisses some as implausible.
- # is both most plausible and most sadistic form of processing inconsistency.
  - To illustrate:

Cognitive Biases

Role of processing inconsistency$\rightleftharpoons$

- BRR show Barney may end up purchasing signals forever,$\rightleftharpoons$
  - with unbounded welfare loss!$\rightleftharpoons$
  - Prospective "acceptive" $\Longrightarrow$ Barney thinks new info can substantially change his beliefs. $\rightleftharpoons$ But
  - retrospective pooling $\Longrightarrow$ impact of additional signal on beliefs small$\rightleftharpoons$
  - Inference driven increasingly by proportion of *a* signals, so marginal impact of a signal approaches zero.$\rightleftharpoons$
  - Barney always believes an additional signal will have the same impact.$\rightleftharpoons$

Now ... let us revisit general categories of biases in belief updating$\rightleftharpoons$

- And general features of updating that might differ from Bayesian$\rightleftharpoons$

Questions?

Updating:

- Bayesian: $\ln(\frac{p_{t+1}}{1-p_{t+1}}) = \ln(\frac{p_t}{1-p_t}) + \ln(\frac{\sigma_t}{1-\sigma_t})$
- Non-Bayesian: $\ln(\frac{p_{t+1}}{1-p_{t+1}}) = \alpha \ln(\frac{p_t}{1-p_t}) + \beta \ln(\frac{\sigma_t}{1-\sigma_t})$

Claim 1: With limited data & examples, researchers may mistake under-reaction to some factor x for over-reaction to factor y.

- Fundamental Theorem of Algebra:

Theorem: $\forall z, x, y, \alpha$ such that $sign(y) \neq sign(x)$, $\alpha \in (0, 1)$, and $z = \alpha x + y$, $\exists \beta > 1$ such that $z = x + \beta y$.

- Proof: Let $\beta = 1 - \frac{(1-\alpha)x}{y}$
- (Of course if have lots of x, y, can identify α, β)

Claim 2: We can associate common errors with patterns of how movement and uncertainty resolution compare to Bayesian:

Cognitive Biases

(Some) Problems with the simple BRN formula:$_{\rightleftharpoons}$

- Similar problems other biases

For all the pretense to do PEEMish modification of Bayes:$_{\rightleftharpoons}$

- Much left out of formulas that matter for updating.$_{\rightleftharpoons}$
- Hard to mechanically apply.$_{\rightleftharpoons}$

No formal difference “no information” *vs* “useless information.”$_{\rightleftharpoons}$

- Possible incremental improvement: if r.v. operates as signal, BRN even realization useless$_{\rightleftharpoons}$

Beliefs depend on hypotheses focused on.$_{\rightleftharpoons}$

- As with other non-Bayesian models, sensitive to “hypothesis-splitting.”$_{\rightleftharpoons}$
- Inference about $\{A, B, C\} \neq$ inference about $\{A \cup B, C\}$.$_{\rightleftharpoons}$

These framing effects are important and true$_{\rightleftharpoons}$

- But make application of BRN sensitive to these framings.$_{\rightleftharpoons}$

Suppose:

- Let $q_A = p(s|A)$, $q_B = p(s|B)$, $q_C = p(s|C)$
- Priors: p_A, p_B, p_C
- Let $q_{A\cup B} = \frac{p_A q_A + p_B q_B}{q_A + q_B}$

Extreme Saki:

- $p(A|s)_{\{A,B,C\}} = \frac{q_A}{q_A+q_B+q_C}$
- $p(B|s)_{\{A,B,C\}} = \frac{q_B}{q_A+q_B+q_C}$
- $p(C|s)_{\{A,B,C\}} = \frac{q_C}{q_A+q_B+q_C}$
- $p(A\cup B|s)_{\{A\cup B,C\}} = \frac{q_{A\cup B}}{q_{A\cup B}+q_C}$

It can be shown that

- $p(A\cup B|s)_{\{A\cup B,C\}} < p(A|s)_{\{A,B,C\}} + p(B|s)_{\{A,B,C\}}$

Indeed ... are we sure can't have:

- $p(A\cup B|s)_{\{A\cup B,C\}} < p(A|s)_{\{A,B\cup C\}}$?

Suppose Gus tells Tommy (Bayes) and Saki (BRN):

- "I was mesmerized last night"

Compare two different questions we could ask Tommy and Saki:

- "Probability Gus saw a Movie Last Night?"
- "Probability Gus saw a Johnny Depp Movie Last Night"

Tommy and Saki share priors before being told mesmerized:

- $prob(JD) = 1\%$,
- $prob(OtherMovie) = 9\%$,
- $prob(NoMovie) = 90\%$

And beliefs of conditional probability of mesmerized:

- $prob(mesmr|JDmovie) = 1.00$,
- $prob(mesmr|otherMovie) = .10$,
- $prob(mesmr|NoMovie) = .10$

How will Tommy and Saki answer the two questions?

Cognitive Biases

If asked Prob(Depp Movie), Tommy says: $\rightleftharpoons$

- $Prob(Depp|mesmerized) = \frac{[1][.01]}{[1][.01]+[.1][.99]} = 9.2\%$ $\rightleftharpoons$

If asked Prob(Movie) (= Depp + Non-Depp), Tommy says: $\rightleftharpoons$

- $Prob(Movie|mesmerized) = \frac{[(1)(.1)+(.1)(.9)][.1]}{[(1)(.1)+(.1)(.9)][.1]+[.1][.9]} = 17.4\%$ $\rightleftharpoons$

If asked Prob(Depp Movie), (Extreme) Saki says: $\rightleftharpoons$

- $Prob(Depp|mesmerized) = \frac{[1][.5]}{[1][.5]+[.1][.5]} = 90.9\%$ $\rightleftharpoons$

If asked Prob(Movie) (= Depp + Non-Depp), (Extreme) Saki says: $\rightleftharpoons$

- $Prob(Movie|mesmerized) = \frac{[(1)(.1)+(.1)(.9)][.5]}{[(1)(.1)+(.1)(.9)][.5]+[.1][.5]} = 65.5\%$ $\rightleftharpoons$

Hmmm.

$\looparrowright$

Cognitive Biases

Final "problem" for BRN is an incompleteness$\rightleftharpoons$

- **Complete models for economics must say DM's beliefs about future updating.**$\rightleftharpoons$
  - Evidence (on most biases) is retrospective$\rightleftharpoons$
  - Only asking what people think after seeing evidence.$\rightleftharpoons$
  - But (e.g., search) must know prospective.$\rightleftharpoons$
- In context of NBLLN, Benjamin, Rabin, and Raymond (2013): $\rightleftharpoons$
  - framework to think about "retrospective" vs "prospective".$\rightleftharpoons$
  - Needn't be consistent$\rightleftharpoons$
  - E.g., Barney prospective separating, retrospective pooling.$\rightleftharpoons$
- Our best guess: $\rightleftharpoons$
  - Saki thinks "prospectively" that she'll pay attention to base rates.