Disclaimer

- These slides are posted solely to accompany Matthew's lectures at the 2016 Russell Sage Summer Institute in Behavioral Economics.

- They are not edited carefully as stand-alone notes, and are not intended for general circulation.

Cognitive Biases

So far we've done:

- "quasi-maximization" models that posit that at each moment in time a person is constrained-maximizing a well-defined utility function—just not the 'right' one.

Now we'll do:

- "quasi-Bayesian" models that posit a person is maximizing utility w.r.t. to beliefs that are in error, as modeled by some specific distortion of Bayesian information processing.

What does the 'quasi' mean? Definitions and synonyms:

- resembling; seeming; virtual; having some, but not all, of the features of; as if.

Cognitive Biases

- Before outlining errors, an aside ...$\rightleftharpoons$

An aside on Managed Funds$\rightleftharpoons$

- Suppose that you observed the following advertisement, in its entirety from a brokerage firm that will advise you on stocks:$\rightleftharpoons$

"We value you, the client."$\rightleftharpoons$

- What would you infer from this statement?

$\looparrowright$

Cognitive Biases

- Without answering, let me make up some statistics (my version of empirical work ...) to make it clear.

Suppose 813 funds in U.S. in business at least the last 2 quarters, and

- .... 202 of them have ads saying
  - "We've beat the market average the last 2 quarters"
- .... 197 of them have ads saying
  - "We beat the market average the last quarter"
- .... 414 of them have ads saying
  - "We value you, the customer"

Now what do you think "We value you, the customer" means?

- It probably means that the mutual fund lost to the market last quarter.

Cognitive Biases

Was my question unfair?⇌

- Maybe, maybe not.⇌
- I didn't give you context/comparison.⇌
- But not at all clear that the mutual fund would give you context.⇌
- Without context, you don't know what to make of it. Maybe salient if you see lots of the other ads; depends.⇌
- Those that have something good to report, report.⇌
- Those who don't, tell you how valuable you are to them.

↬

Cognitive Biases

Issues of what you pay attention to, including the strategic logic of situations, the role of "dogs not barking", etc., may play a huge role in inference.

- Not just whether you use appropriate Bayesian updating or appropriate strategic logic given the information you focus on
- But whether you focus on all the questions you should focus on.
- We'll get at a bit at end.

- But this wasn't really the topic I wanted to pursue

Suppose that all 414 of them saying they value you lost to the market average last quarter.

- An easy question to segue into next topic:

What obvious inference would you make from the three different ads.

- What is the quality of a fund that advertises it has beat the market two quarters in a row vs. the other two categories?

Cognitive Biases

Probably ... they are of the same quality. They are all probably average.$\rightleftharpoons$

- Why?$\rightleftharpoons$

Very close to the distribution you'd get if they were all average.$\rightleftharpoons$

- Randomly 25% would beat market twice in a row, 50% would lose the last quarter.$\rightleftharpoons$

I haven't given much to go on, and probably there is a clever way to infer difference in quality.

- But I'd argue these statistics are suggestive that all funds are average.

$\looparrowright$

Cognitive Biases

What if you did not easily know the distribution of performances?

- What should you infer from seeing good recent performance?
- Should you pay to get good evidence on recent performance?

We happen to know such distributions are true.

- But what if investors don't?

It could be argued (and it would be correct) that they *should* know

- Logic of financial markets should tell them not to search for patterns

Final topics of course related to that.

- Will turn out (among other lessons):
  - Often see patterns where there are none (cognitive biases),
  - and where you should know a priori that there are none! (cursedness)

Enough of me doing finance.

Cognitive Biases

Outline:

1. Modeling Cognitive Errors
2. LSN/NBLLN/SSN in sample prediction
3. Bin Effects
4. Disentangling NBLLN and Bin Effects
5. LSN/NBLLN/SSN in inference
6. Base-Rate Neglect
7. Things it would be useful to have more data on

$\looparrowright$

Cognitive Biases

Today's talk based on:

- Two were-surely-going-to-be-finished-by-now unfinished papers
  - "Base-Rate Neglect" with Dan Benjamin and Aaron Bodoh-Creed
  - "Misconceptions of Chance: Evidence from an Integrated Experiment," with Dan Benjamin and Don Moore (previous draft on line)
  - "Belief Movement, Uncertainty Reduction, and Rational Updating," with Ned Augenblick
- Some existing papers of mine:
  - "Inference By Believers in the Law of Small Numbers"
  - "The Gambler's and Hot-Hand Fallacies" with Dimitri Vayanos
  - "A Model of Non-Belief in the Law of Large Numbers," with Dan Benjamin and Collin Raymond
- And mostly:
  - Work above and today based on lots and lots of existing research ... **Griffin and Tversky**, Tversky and Koehler, gobs of Kahneman, Tversky, others.

Cognitive Biases

Approach building from research under the broad heading of “judgment and decisionmaking" (JDM)⇌

- How people’s probabilistic judgments might be distorted.⇌
- Probabilistic reasoning *not* random or totally irrational.⇌
- Human rationality, not superhuman rationality or subhuman idiocy.⇌

Types of formal bias models:⇌ “Misfunctional Bayesian”: ⇌

- Use conditionals and priors, but with wrong functional form.⇌
    - Base-rate neglect.⇌

But many models of errors stick closer:⇌ Quasi–Bayesian Models⇌

- Assume people engage in putatively proper Bayesian updating.⇌
- But specify a precise way in which they either mis-observe or mis-understand how that evidence relates to the hypotheses.⇌
- Examine the implications of Bayesian updating given the error.

Cognitive Biases

- Two categories of quasi-Bayesian $\rightleftharpoons$

Warped-Model Bayesian: $\rightleftharpoons$

- False (but internally consistent) model of how signals are generated. $\rightleftharpoons$
  - Barberis-Shleifer-Vishny (1998), $\rightleftharpoons$
  - Rabin (2002), Rabin and Vayanos (2010), $\rightleftharpoons$

Information-Misreading Bayesian: $\rightleftharpoons$

- Right model, but misread signals. $\rightleftharpoons$
  - Rabin and Schrag (1999) $\rightleftharpoons$
  - Mullainathan (2002).

$\looparrowright$

Cognitive Biases

Different combinations of bias and environment can lead to:

- Overinference/overconfidence ... infer "too much" from information.
- Underinference/underconfidence ... people infer too little

These are **manifestations** of biases, not types of biases.

- Psychologists first, now economists: talk like direction of beliefs vs. appropriate Bayesian are general tendencies.
- Now too many researchers pitch generic "Overconfidence"/overinference
- There is no general tendency towards over- or under-inference
  - And no great definition outside of binary questions

Sampling Biases$\rightleftharpoons$

Now review just subset of biases$\rightleftharpoons$

- **The** (!) important biases in the simplest setting:$\rightleftharpoons$
- Take an *i.i.d.* process *should* know is *i.i.d.*, or at some level *do* know.$\rightleftharpoons$
- Prototype: (possibly biased) coin yields h in proportion $\theta \in (0, 1)$.$\rightleftharpoons$
  - What are beliefs about samples given θ?$\rightleftharpoons$
  - What do people infer about θ from samples?

$\looparrowleft$

Cognitive Biases

In coins & urns, essentially 3 biases:$\rightleftharpoons$

- Law of Small Numbers and Gambler's Fallacy:$\rightleftharpoons$
  - Tversky and Kahneman (1971): exaggerated belief that small samples and short streaks will reflect population mean.$\rightleftharpoons$
- Non-belief in LLN:$\rightleftharpoons$
  - Very large psychology literature (1965ish-1975ish) on "conservatism."$\rightleftharpoons$
  - BRR (2012) meta-analysis: under-inference from non-small samples.$\rightleftharpoons$
- Base-Rate Neglect$\rightleftharpoons$
  - Underweighting priors when processing new information

$\looparrowright$

Cognitive Biases

LSN, GF, and NBLLN are, as primitives, distortions in beliefs about likelihood of different samples being generated from given θ.

- Bias: instead of Bayesian $p(s|\theta)$, some $\widetilde{p}(s|\theta)$.
  - Rabin (2002), RV (2010), & BRR (2012)
- These theories predict misinference from assuming

$$p(\theta|s) = \frac{\widetilde{p}(s|\theta)p(\theta)}{\sum_{\theta'} \widetilde{p}(s|\theta')p(\theta')}.$$

- Ignore BRN for now
- Seems like is close to true:
  - *Except* for BRN, inference errors more or less accord with Bayesian inference applied to sample-prediction errors.

Now: evidence on ways $\widetilde{p}(s|\theta) \neq p(s|\theta)$.

- GF/LSN, NBLLN $\rightarrow$ SSN
- We'll come back to inference problems.

Cognitive Biases

Gambler's Fallacy

- Evidence in BMR (2013, 2016)
- But better evidence from earlier:
- Maryland State Pick-Three Lottery
  - Pari-mutuel Betting provides good data: Infer numbers from odds created, AND have people losing money from bad beliefs (many prediction tasks i.i.d. world means no wrong behavior).
  - Bet 50 cents on day's 3-digit draw, and winners get 52% of total bets (this is typical state cut).
  - If $\frac{1}{10}$% people bet on a number, it pays \$260. More than \$260 means $< \frac{1}{10}$% bet on it; less than \$260 means $> \frac{1}{10}$%.

Cognitive Biases

Terrel (1994) reported average winnings as function of when the last time that number won

Within Week:	*$349*
1-2 Weeks ago:	*$349*
2-3 Weeks ago:	*$308*
3-8 Weeks ago:	*$301*
Not Within 8 Weeks:	*$260*
Overall:	*$262*

E.g., 25% fewer bet on number if won in last 2 weeks. Expected return 34% higher betting on recent winners than recent losers.

What do people predict about samples?

KT (1973): How likely different proportions heads in a 50/50 coin?

		45-55%	75-85%
$N = 10$	true	25%	4%
	people think	20%	6%
$N = 100$	true	68%	$\approx$ 0%
	people think	22%	5%
$N = 1000$	true	$\approx$ 100%	$\approx$ 0%
	people think	21%	5%

Cognitive Biases

- Already by N = 10, distribution too dispersed.⇌
  - By N = 1000, it is extreme.⇌
- Now: BMR (2013,2016) replicate, get very similar results, but incentivized and frequentist.⇌
- But we have design feature that changes interpretation of some⇌
- Also, evidence for GF won't discuss today⇌
  - (After 9 heads, people expect $\frac{2}{3}$ chance tails.)⇌

Posted 2013 version, but now report updated data too. ⇌

- But first, aside central to our experiment

↬

Support Theory and Bin Effects

- (Close cousin to stuff probably very familiar to you)
- Per Tversky and Koehler (1994) and other models, small probability events exaggerated, and combining events reduces total weight.
- Suppose elicit beliefs p on exhaustive/exclusive 3 events $\{A, B, C\}$,
    - $p(A) + p(B) + p(C) = 1.$
- And elicit q on 2 events $\{A \cup B, C\}$,
    - $q(A \cup B) + q(C) = 1.$
- Then will find: $q(A \cup B) < p(A) + p(B)$ and $q(C) > p(C).$
    - **Important corollary: tendency to overestimate small probabilities**

BMR experiment:

- Skipping details on design.
  - Posted and future paper has details
- 100 subjects Pittsburgh food court, 300 subjects Berkeley experimental lab

Motivation for Experiment:

- Incentivized evidence relatively sparse. Especially for NBLLN.
- Eliminate confounds one might sensibly worry about.
- Eliminate confounds one might non-sensibly worry about.
- Triangulation
  - **Within-subject inconsistencies on same data ... even exotic beliefs about experimental design can't explain.**

- In shared introduction to most questions, we told subjects:
  - "We flipped a coin ten times. Actually, we had a computer simulate the coin flipping, generating exactly the same type of random series real coins do. This was a fair coin, in the sense that coin could come up either heads or tails, and there was an equal chance of each. That generated one ten-flip set. Then we did it again, and again, and again, until...we had 1 million ten-flip sets."
- Generated 1 million samples each size using the Matlab randomizer.
- **All questions elicit beliefs regarding (single) generated sample.**
- Entirely within-subject design

Histograms

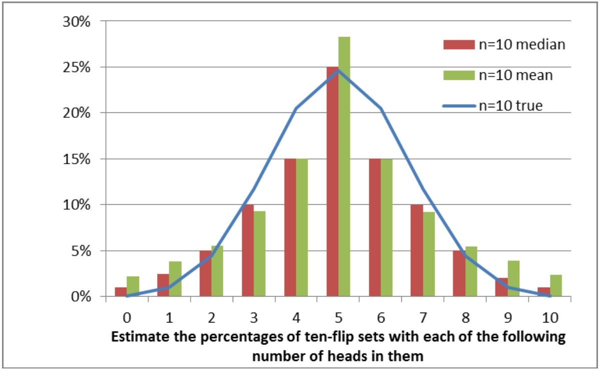
- Eliciting beliefs about the frequency distribution of outcomes.
- Subjects typed in a number between 0 and 100 for each group.
- Screen showed the sum of the percentages.
- Required sum = 100% before could continue to next screen.

Cognitive Biases

- NBLLN?$\rightleftharpoons$ ... (Once controlled for bin effects?)$\rightleftharpoons$
  - Basic answer: Yes, for 1,000 and 1,000,000$\rightleftharpoons$
  - **Not** too dispersed for $N = 10$, once bin effects controlled for$\rightleftharpoons$
  - Confirm KT SSN claim: no differences 10, 100, 1k, 1m$\rightleftharpoons$
  - But shockingly little rebellion against CLT in 1,000,000 case! ... $\rightleftharpoons$
    - Non-belief in LLN rather than disbelief in LLN?
- Clear and Big "Bin Effects"$\rightleftharpoons$
  - Beliefs 5/10 heads from 20% to 36% by binning more coarsely;$\rightleftharpoons$
  - Beliefs on 451-549 / 1000 heads from 19% to 40%.$\rightleftharpoons$
- Were indeed confounding results$\rightleftharpoons$
- Smoking-gun approach:$\rightleftharpoons$ **Within partition:** $\rightleftharpoons$
  - when right answer $\pi(A) \leq \pi(B)$, subjects believe $p(A) > p(B)$$\rightleftharpoons$
  - then bias towards A.$\rightleftharpoons$
  - (Dan Benjamin and I working on more general exploration of method).

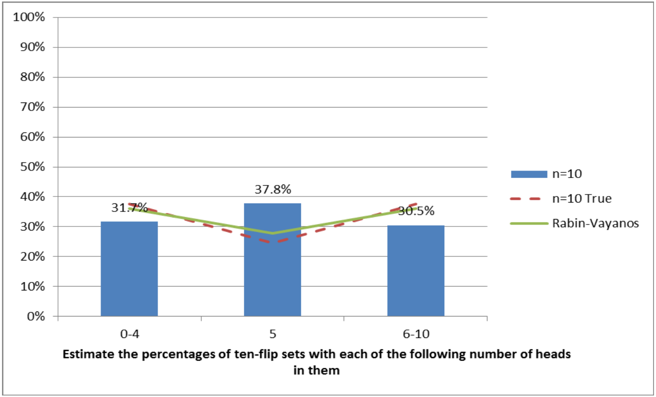
Cognitive Biases

We get “exact representativeness”, but else too spread:



- Just like KT earlier results.
- But the "too spread" might just be probability compression

But re-binning reverses the results:⇌

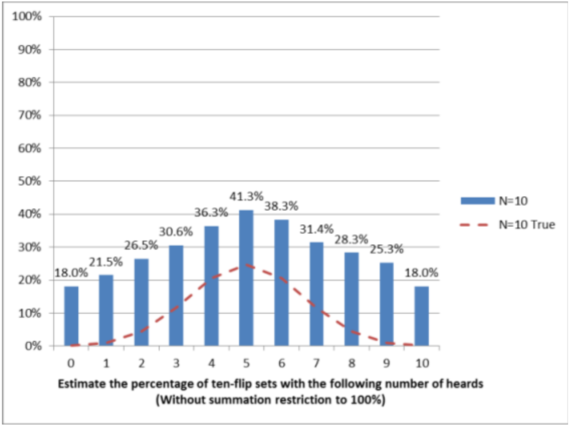


⇌

- Note the smoking-gun evidence: ⇌
    - We know tails too thin because distortion relative to truth opposite of compression

Proof that compression is a confound:

- When done separately, all less-than-50% bins get exaggerated.
- "What is probability of N heads?"



So N=10 over-dispersion bad interpretation.$\rightleftharpoons$

- But what about large samples?$\rightleftharpoons$

NBLLN (sample sizes 1,000 and 1,000,000)$\rightleftharpoons$

- With same eleven-outcome, confounded-with-compression approach, over-dispersion for N=1,000:

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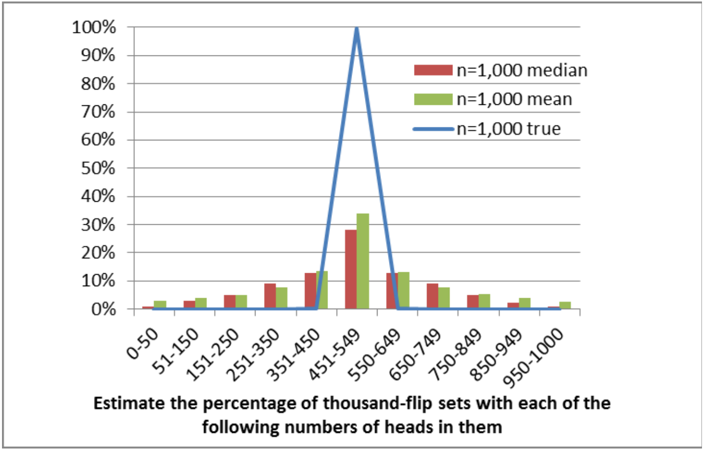
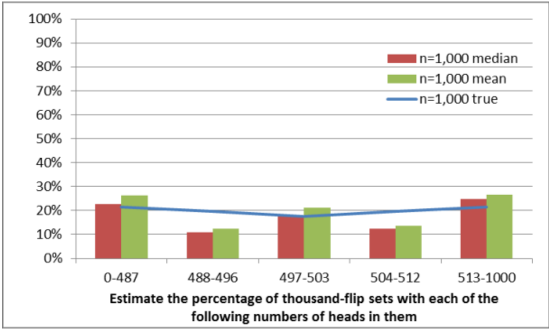
Cognitive Biases



But we can show **not** just binning:⇌

- 5-equal-bins treatment nails it⇌



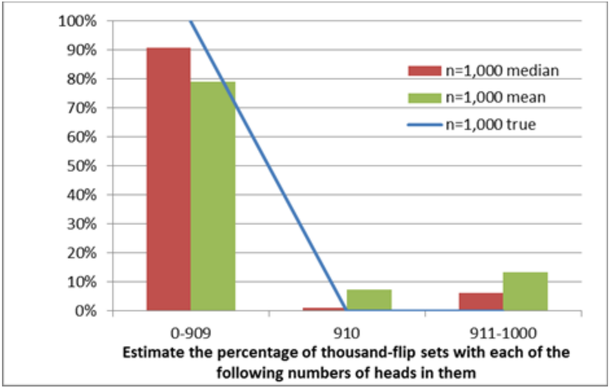
⇌

- "W" may be real representativeness.⇌
- But clear outer tail vs. inner tail result.⇌

My favorite example shows real under-appreciation of CLT:

Cognitive Biases

Probabilities {0-909,910,911-1000}?⇌ Mean {79%,7%,14%}, Median {91%,1%,6%}



⇌

- People don't realize that 911-1,000 never happens.

↬

Cognitive Biases

But do even *we* appreciated thinness of tails? $\rightleftharpoons$

- True fact: $p(910) > 5 \cdot p(911 - 1000)$. $\rightleftharpoons$ (Who knew?) $\rightleftharpoons$
- (Question to contemplate: is the bell curve as optical illusion?) $\rightleftharpoons$

How is our intuition on CLT/LLN? $\rightleftharpoons$

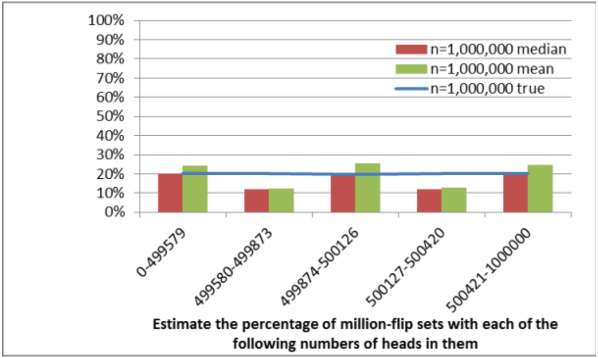
- Probability that
  - Exactly 60% of 40 flips heads? $\rightleftharpoons$
  - Exactly 90% of 40 flips? $\rightleftharpoons$
  - Both very small ... $\rightleftharpoons$ but how small? How compare? $\rightleftharpoons$
- Betting your intuition is bad / off. What is $\frac{prob(36/40)}{prob(24/40)}$? $\rightleftharpoons$
  - About $\frac{1}{680{,}000}$. $\rightleftharpoons$
  - Not knowing exact $\longrightarrow$ "bound error"; $\rightleftharpoons$
  - finding this shocking $\longrightarrow$ "astray error".

$\looparrowright$

Cognitive Biases

Also elicited beliefs about sample size 1,000,000

- First time ever such beliefs have been elicited?
- DB and MR astonished by results. **“Nuance”** not reporting today
- Non-belief in LLN we think is right, but vigorous disbelief less true than we thought?



Cognitive Biases

Outline of last two lectures:

1. More cognitive biases
    1. LSN/NBLLN/SSN in inference
    2. Base-Rate Neglect
    3. Things it would be useful to have more data on
2. Failures of inference from volitional agents
    1. "Cursedness": inferring too little from others' actions
    2. Redundancy neglect: attending too little to the redundancy in others' actions

↬

Cognitive Biases

LSN, GF, and NBLLN as primitives in distortions in beliefs about likelihood of different samples being generated from given θ.

- These theories predict misinference from assuming

$$p(\theta|s) = \frac{\widetilde{p}(s|\theta)p(\theta)}{\sum_{\theta'} \widetilde{p}(s|\theta')p(\theta')}.$$

Now: evidence on LSN, NBLLN $\rightarrow$ SSN in inference problems.

- Ignore BRN for now

Evidence on LSN, NBLLN, & SSN in Inference

Beautiful experiment: Griffin and Tversky (1992).

- "Imagine that you are spinning a coin, and recording how often the coin lands heads and how often the coin lands tails. Unlike tossing, which (on average) yields an equal number of heads and tails, spinning a coin leads to a bias favoring one side or the other because of slight imperfections on the rim of the coin (and an uneven distribution of mass). Now imagine that you know that this bias is 3/5. It tends to land on one side 3 out of 5 times. But you do not know if this bias is in favor of heads or in favor of tails."

Tempting model to capture both: Sample-Size Neglect.⇌

- Kahneman & Tversky (1973) and Griffin and Tversky (1992):⇌
  - People attend to proportions, not sample size.⇌
  - Intuitive model, **not far off**.

↬

Cognitive Biases

Two possible $i.i.d.$ coins each with prob $= .5$: $\rightleftharpoons$

- $\pi(h|\cdot) = \frac{3}{5}$ coin and a $\pi(h|\cdot) = \frac{2}{5}$ coin$\rightleftharpoons$
- Observe a set of flips h, t.$\rightleftharpoons$
- Bayes' Law says $\frac{\pi(\theta|h,t)}{\pi(1-\theta|h,t)} \equiv l(h,t) = \left(\frac{3}{2}\right)^{h-t}$.$\rightleftharpoons$
- So Bayesian inference from (h, t) depends solely on $h - t$.$\rightleftharpoons$
- That's all well and good ... but:$\rightleftharpoons$
  - Only geeks think that way.$\rightleftharpoons$
  - And only when paying attention.$\rightleftharpoons$
  - People in fact base beliefs on how close $\frac{h}{h+t}$ looks to $\frac{3}{5}$ vs. $\frac{2}{5}$.$\rightleftharpoons$

A tale of two tables **presenting the same data**:

$\looparrowright$

Over-infer from small samples, under-infer from large samples. $\rightleftharpoons$

Sample of (h,t)	$h+t$	$h-t$	Median $P(\theta = \frac{3}{5} \mid h, t)$	Proper $B(\theta = \frac{3}{5} \mid$
5,0	5	5	.92	.88
7,2	9	5	.77	.88
11,6	17	5	.64	.88
19,14	33	5	.60	.88
3,0	3	3	.85	.77
4,1	5	3	.80	.77
6,3	9	3	.67	.77
10,7	17	3	.60	.77
2,1	3	1	.63	.60
3,2	5	1	.60	.60
5,4	9	1	.55	.60
9,8	17	1	.54	.60

↬

Cognitive Biases

Meta-Lesson: **proportional thinking**$_{\rightleftharpoons}$

Sample of (h,t)	% heads	Median $P(\theta = \frac{3}{5} \mid h, t)$	Proper $B(\theta = \frac{3}{5} \mid h, t)$
5,0	100%	.92	.88
3,0	100%	.85	.77
4,1	80%	.80	.77
7,2	78%	.77	.88
6,3	67%	.67	.77
2,1	67%	.63	.60
11,6	65%	.60	.88
3,2	**60%**	**.60**	**.60**
10,7	**59%**	**.60**	**.77**
19,14	**58%**	**.60**	**.88**
5,4	55%	.55	.60
9,8	53%	.54	.60

$\looparrowright$

Meta-Lesson: **proportional thinking**$_\rightleftharpoons$

- Basing on proportions leads to “Sample-Size Neglect”:$_\rightleftharpoons$

Under-using sample size.$_\rightleftharpoons$

- If double h and t, double $h - t$
- hence change a Bayesian’s inference.$_\rightleftharpoons$
- But you won’t change $\frac{h}{h+t}$.$_\rightleftharpoons$
  - So people infer same.$_\rightleftharpoons$
- **Over-infer from small samples, under-infer from large samples.**$_\rightleftharpoons$
- Discussion in BRR (2012) why don’t model just as "SSN"$_\rightleftharpoons$
- **Note:** sometimes underinference, underconfidence

More attitude problem! $\rightleftharpoons$

- Is thinking in terms of $h - t$ System 1 or System 2? $\rightleftharpoons$
- Is thinking in terms of $\frac{h}{h-t}$ System 1 or System 2? $\rightleftharpoons$
- And if we knew Sissy were using System X... $\rightleftharpoons$
    - (whichever is the bad one) $\rightleftharpoons$
        - Would we know how she uses sample size? $\rightleftharpoons$

Whatever its conceptual usefulness and neuro-truth of system thinking... $\rightleftharpoons$

- Severe danger of false consciousness. $\rightleftharpoons$
- And empirical complacency. $\rightleftharpoons$

Cognitive Biases

Some economic implications?

Implications of LSN? $\rightleftharpoons$

- Rabin (2002) models: false belief by investors in the value of advice/managed funds may be LSN-related. $\rightleftharpoons$
- People over-infer from short-run performance of mutual fund that its manager must be a genius. $\rightleftharpoons$
- "Fictitious Variation": $\rightleftharpoons$
  - When looking at global data, in fact might infer skill out there in investing when there is none. $\rightleftharpoons$
- Rabin and Vayanos (2010) also explain other investment errors, such as under- and over-reaction and false belief in hot hands. $\rightleftharpoons$
- More generally: How and when GF $\longrightarrow$ HH.

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Implications of NBLLN?

- People unconvinced by statistics.
- And: NBLLN is underemphasized as a *necessary* enabler and almost-sure confound for "over-extraction" biases.
  - NBLLN says under-infer from Consumer Reports data sets.
  - Salience/vividness: infer too much from friend's bad (& costly) experience.
  - But: even if over-weighted your friend by 500 times, still follow *Consumer Reports* if believe in LLN.
  - Even the greats have been sort of mis-emphasizing their own results.
- Exacerbates risk/loss aversion:
  - People exaggerate chance of losing from large number of better-than-fair bets.
  - Not just about narrow bracketing.

Applying NBLLN to Compound Risk

- Samuelson's colleague said he would reject a 50/50 bet to gain $200/lose $100, but accept 100 independent repetitions.
- Samuelson's (1962) theorem: Absent implausible wealth effects, a Tommy with expected-utility preferences over wealth takes $N \geq 1$ repetitions of a bet if and only if he will take one.
  - Intuition: If no wealth effects, EU/DMU(W) says prefer $k+1$ over k iff prefer 1 to 0. So iff prefers 1 to 0 he prefers 2 to 1 and 3 to 2... so prefers 100 to 0.
- NBLLN does **not** explain Samuelson's colleague.
  - Samuelson's theorem *does* extend to Barney! An EU/DMU(W) Barney who rejects one gamble will reject many independent plays. Probably more so than Tommy!

- Indeed, the point lies elsewhere ...
- Of course Samuelson's colleague was *not* an EU/DMU(W) guy.
  - A trait he shares with billions ...
  - Calibrationally, that degree of risk aversion must be about *loss aversion* or some other reference-dependent cousin of loss aversion.
- He was also clearly right to want to take the 100 bets: it has expected gain of $5,000, the chance of a net loss is only 1/700, and the chance of losing more than $1,000 was only 1/26,000.
- Most people would take the 100 independent bets.
  - Loss aversion plus narrow bracketing together can explain SC
  - You'd have to be insane not to take that cumulative bet.
  - And yet ...

Cognitive Biases

- Many people also turn down 100 repetitions!

For the same (hypothetical) question, Benartzi and Thaler (1999) find:

- Evening MBA students: 64% take single, only 75% take 100 repetitions.
  - Huh?
- Coffee shop visitors: 43% take single, only 66% take 100 repetitions.
  - Huh?
- Undergraduates: 77% take single, only 50% take 100 repetitions.
  - Huh???

So why do so few people agree with Samuleson's Colleague?

- Answer: they're not as smart as Samuelson's Colleague

Cognitive Biases

Enter Barney... people massively overestimate chance of losing money.

- 49% of undergrads take 150 repetitions of real-stakes gamble of 90% win 10 cents, 10% lose 50 cents.
- This has expected gain of $6. And less than 1/300 chance of losing money.
- But: B&T asked subjects likelihood of losing money.
  - Answer: 24%!
  - not because exaggerating bad outcomes ... similar results when reverse.
- Mis-estimation of probabilities appears to drive behavior:
  - when shown the histogram, proportion of undergrads taking the repeated bet goes from 49% to 90%.
- We think it is clear that this isn't "narrow bracketing" in way usually thought of. It is all about Barney.

Cognitive Biases

Calibrationally, Barney goes much of the way toward explaining why 25-57% of people turn down 100 repetitions of 50/50 bet to gain \$20/lose \$10.

- But not far enough: 100 repetitions yields exp(gain)/exp(loss) $>$ 32,000.
- Tommy realizes this.
- But 10-Barney thinks exp(gain)/exp(loss) $>$ 15. So with reasonable loss aversion would also take.
- (Fatter tails would reduce the favorability further.)
- Analogously, NBLLN leads people to overestimate the likelihood of equities losing money in the long run, and why people prefer to invest more in equities when shown the true distribution of returns.

Implications of Bin Effects?

- Is it just an elicitation confound?
  - To ID nature of bias, we "control" for it.
  - But not clear that there is any such thing as "true" preference, independent from bin effects.
  - And, presumably, also a real thing economically.
- If people act as if the total probability of cancer is higher when broken down by different types, then ...
  - Means structure of doctor communication, insurance, etc., matter.

Now: Base-Rate Neglect

- Model (stated, estimated over the years):
- Base-rate-neglect sufferer (**Saki**) believes:

$$p(\theta|s) = \frac{p(s|\theta)p(\theta)^{\alpha}}{\sum_{\theta' \in \Theta} p(s|\theta')p(\theta')^{\alpha}}.$$

- where $\alpha \in [0, 1)$. Extreme $\alpha = 0$ Saki:

$$p(\theta|s) = \frac{p(s|\theta)}{\sum_{\theta' \in \Theta} p(s|\theta')}.$$

- Combine LSN, GF, and NBLLN with BRN:

$$p(\theta|s) = \frac{\widetilde{p}(s|\theta)p(\theta)^{\alpha}}{\sum_{\theta'} \widetilde{p}(s|\theta')p(\theta')^{\alpha}}.$$

Cognitive Biases

The textbook example of BRN:$\rightleftharpoons$

- Test for disease is 90% accurate (symmetrically)$\rightleftharpoons$
- 5% of tested population have disease$\rightleftharpoons$

If test positive, probability of disease?$\rightleftharpoons$

- Stats text: tells you right answer (32%)$\rightleftharpoons$
- Psych articles: shows people give wrong answer (say, 90%).$\rightleftharpoons$

But ... $\rightleftharpoons$

- You might be left with wrong impression from BRN ...

$\looparrowright$

Cognitive Biases

Test	Priors	Posteriors		Frequency
		Tommy	Saki	
Positive	5%	32%	90%	14%
Negative	5%	$< 1\%$	10%	86%

- Yes, her beliefs have moved too much
  - Tommy beliefs moved (up) 27% or (down) 4%.
  - Saki's beliefs moved (up) 85% or (up) 5%.
- But, while Saki beliefs too extreme after positive result ...
  - (which is the main image of BRN)
  - But too **moderate** after negative result
- Turns out a general feature of BRN:
  - Beliefs move too much, but are too moderate
  - In dynamic, i.i.d. model, ergodic non-convergence.

Cognitive Biases

- In fact, a bizarre "moderation effect":
  - buried in definition, buried in literature

Test	Priors	Posteriors		Frequency
		Tommy	Saki	
Positive	5%	32%	90%	14%
Negative	**5%**	$< 1\%$	**10%**	86%

- After **negative** signal, her beliefs move **up.**
- Wouldn't necessarily bet on that implication here.
  - But it is an implication that the model must own ...
  - And just maybe wants to own ...

Cognitive Biases

Griffin and Tversky (1992) also best evidence on BRN? $\rightleftharpoons$

- One of very few examples where signal, base rates same direction $\rightleftharpoons$
- And, they find the extreme moderation effect ... $\rightleftharpoons$
    - Subjects' with priors 90% chance heads biased, given sample of $(6h, 4t)$, posteriors $< 90\%$.

$\looparrowright$

Cognitive Biases

Fleshing out model in dynamic contexts.

- **When signals arrive sequentially**, cumulative updating?
  - Combine all signals together, then combine with original base rate?
  - Or sequentially update—each new signal generates new base rate?
- We assume 2^{nd}—central to many results.
  - so today's posteriors are tomorrow's priors.
- Data observed in the recent past matters more than data from distant past (even though signals i.i.d.).
  - So, obviously, beliefs from same information depends on order.
  - Forever and ever.
  - As if current day's temperature affects belief in global warming?
    - (Yi, Johnson, and Zaval, 2011).

Cognitive Biases

Implications:

- Recency effect: more recent data matters more.
- Long-run beliefs ergodic
    - initial prior ceases to matter.
    - *Support* of ergodic distribution independent of true state.
    - Never become fully confident.
- Moderation effect: strong priors always dampened.
- Range of these beliefs independent of truth.
    - But frequency of different beliefs does depend on it.
- Interesting implications in reputation context.
    - Perpetually gain and lose reputation
    - No eventual decay.

Cognitive Biases

(Some) Problems with the simple BRN formula:

- Similar problems other biases

For all the pretense to do PEEMish modification of Bayes:

- Much left out of formulas that matter for updating.
- Hard to mechanically apply.

No formal difference "no information" *vs* "useless information."

- Possible incremental improvement: if r.v. operates as signal, BRN even realization useless

Beliefs depend on hypotheses focused on.

- As with other non-Bayesian models, sensitive to "hypothesis-splitting."
- Inference about $\{A, B, C\} \neq$ inference about $\{A \cup B, C\}$.

These framing effects are important and true

- But make application of BRN sensitive to these framings.

Cognitive Biases

Suppose Gus tells Tommy (Bayes) and Saki (BRN):

- "I was mesmerized last night"

Compare two different questions we could ask Tommy and Saki:

- "Probability Gus saw a Movie Last Night?"
- "Probability Gus saw a Johnny Depp Movie Last Night"

Tommy and Saki share priors before being told mesmerized:

- $prob(JD) = 1\%$,
- $prob(OtherMovie) = 9\%$,
- $prob(NoMovie) = 90\%$

And beliefs of conditional probability of mesmerized:

- $prob(mesmr|JDmovie) = 1.00$,
- $prob(mesmr|otherMovie) = .10$,
- $prob(mesmr|NoMovie) = .10$

How will Tommy and Saki answer the two questions?

Cognitive Biases

If asked Prob(Depp Movie), Tommy says:$\rightleftharpoons$

- $Prob(Depp|mesmerized) = \frac{[1][.01]}{[1][.01]+[.1][.99]} = 9.2\%$$\rightleftharpoons$

If asked Prob(Movie) (= Depp + Non-Depp), Tommy says:$\rightleftharpoons$

- $Prob(Movie|mesmerized) = \frac{[(1)(.1)+(.1)(.9)][.1]}{[(1)(.1)+(.1)(.9)][.1]+[.1][.9]} = 17.4\%$ $\rightleftharpoons$

If asked Prob(Depp Movie), (Extreme) Saki says:$\rightleftharpoons$

- $Prob(Depp|mesmerized) = \frac{[1][.5]}{[1][.5]+[.1][.5]} = 90.9\%$$\rightleftharpoons$

If asked Prob(Movie) (= Depp + Non-Depp), (Extreme) Saki says:$\rightleftharpoons$

- $Prob(Movie|mesmerized) = \frac{[(1)(.1)+(.1)(.9)][.5]}{[(1)(.1)+(.1)(.9)][.5]+[.1][.5]} = 65.5\%$ $\rightleftharpoons$

Hmmm.

$\looparrowright$

Cognitive Biases

Final "problem" for BRN is an incompleteness

- **Complete models for economics must say DM's beliefs about future updating.**
  - Evidence (on most biases) is retrospective
  - Only asking what people think after seeing evidence.
  - But (e.g., search) must know prospective.
- In context of NBLLN, Benjamin, Rabin, and Raymond (2013):
  - framework to think about "retrospective" vs "prospective".
  - Needn't be consistent
  - E.g., Barney prospective separating, retrospective pooling.
- Our best guess:
  - Saki thinks "prospectively" that she'll pay attention to base rates.

Role of processing inconsistency$_{\rightleftharpoons}$

- BRR show Barney may end up purchasing signals forever,$_{\rightleftharpoons}$
  - with unbounded welfare loss!$_{\rightleftharpoons}$
  - Prospective "acceptive" $\Longrightarrow$ Barney thinks new info can substantially change his beliefs. $_{\rightleftharpoons}$ But
  - retrospective pooling $\Longrightarrow$ impact of additional signal on beliefs small$_{\rightleftharpoons}$
  - Inference driven increasingly by proportion of *a* signals, so marginal impact of a signal approaches zero.$_{\rightleftharpoons}$
  - Barney always believes an additional signal will have the same impact.$_{\rightleftharpoons}$

$\looparrowright$

Cognitive Biases

BRN is best illustration of: **relative** "overweighting" of something may come from underweighting of something else.

- As with friend vs. consumer reports,
- And (we'll see) own info vs. market prices
- So too with new information vs. priors ...

Claim: With limited data & loose conceptualization, researchers mistake under-reaction to some factor x for over-reaction to factor y.

- Observe: Two bits of info about binary question going in opposite directions $\rightarrow$ log-likelihoods have opposite signs.
- **Fundamental Theorem of Algebra**: For all z, x, y, α such that $sign(y) \neq sign(x)$, $0 < \alpha < 1$, and $z = \alpha x + y$, there exists $\beta > 1$ such that $z = x + \beta y$.
- Proof: Let $\beta = 1 - \frac{(1-\alpha)x}{y}$
- (Of course if have lots of x, y, can identify α, β)

Cognitive Biases

Things it would be useful to have data on.

- We should gather direct evidence on beliefs across domains.
  - Is happening some
  - Should happen more
  - **Will** happen more
- For identifying biases, we need histograms.
  - So please help
- Help us understand how widespread lab errors are in world
  - Evidence is on coins & urns, but want evidence on realer things.
- People's beliefs about how others would interpret given information.
  - Directly gets at issues of interpersonal thinking.
  - But also indirectly at nature of biases, especially confirmatory.
- **Prospective beliefs**
  - Huge: beliefs about future updating
  - Virtually unmentioned in the psychology
  - We've been making models without any evidence

Models of Inference and Learning from Others⇌

Others' behavior might reveal information to us.⇌

- How good gleaning information from others?⇌
- What systematic errors?⇌
- What effects of these errors?⇌

Earlier:⇌

- Failures of statistical reasoning.⇌

Now: ⇌

- Errors in inference from volitional agents, per se

↬

Inference and Learning from Others

What mean "volitional agents per se"?⇌

- In fact, Behavioral GT weird history...
  - Mistakes in game any different than elsewhere?⇌
  - When see sample of independent "people flips" may make same errors as when see sample of independent coin flips. ⇌

Volitional agents: we can (or should!) use our understanding of their motives as additional source of information.⇌

- What mistakes we make in that form of reasoning?⇌

↬

Inference and Learning from Others

Will tell you about two facts about your interactions with others:

1. Their behavior often contains information relevant to you which you do not glean in other ways.
2. Information in two or more people, when they themselves interact, will be correlated by the logic of their interaction.
    - Because they themselves are gleaning information.

Eyster and Rabin propose:

- We tend to under-attend to info content.
- And tend to under-attend to correlation.

Lots of things in the world contain information!

- We often neglect them
- Often see it when not there!

Inference and Learning from Others

Cursed Thinking:

- People under-infer information from others' behavior.
  - Winner's curse in common-values auctions:
  - Lemons and **financial markets**
  - No no-trade results—naturally and directly and disciplinedly

Naive Inference:

- Insofar as *do* attend to information in others' behavior, tend to take at face value.
  - People don't attend to *redundancy* in social beliefs.
- In observational learning:
  - Rationality predicts some imitation ...
    - but typically predicts anti-imitation too
    - and *precludes* extensive imitation without anti-imitation
  - naive inference, redundancy neglect $\Longrightarrow$ ubiquitous imitation
  - overconfidently wrong social beliefs

Topics

1. Cursedness
   1. Motivation and evidence
   2. Implications in asset markets
   3. Compared to (overconfident) "agreeing to disagree".
2. Rational and Naive Observational Learning
   1. **Behavioral** implications of rational observational learning
   2. Informational, societal consequences of naive herding

Inference and Learning from Others

"Cursedness"⇌

- Eyster and Rabin (2005) "cursed equilibrium":⇌
  - From "Winner's Curse" in Auctions:⇌
  - Bid your estimate and win $\rightarrow$ others more negative information.⇌
  - People don't fully think through the informational content in others' behavior.⇌
- **Interpretation: Limited Rationality**,⇌
  - **As if** think others' behavior independent of their information?⇌
    - Because eqbm, would be very strange.⇌
  - **Not** interpreted thusly.⇌
  - **Rather:** inattentiveness to the fact that they are doing so.⇌
- Strong (& tenuous) feature:⇌
  - correctly predict others' average *behavior*.⇌
- **Portable and pinned-down** model of "information neglect"⇌
  - Formalism misses lots⇌
  - (including lots of psych that is part of motivation)

Winner's curse in trade/lemons: Example

- Firm has book value $v \sim U[0,1]$.
- Firm knows book value, Raider does not.
- Firm values at v and Raider at $\frac{3}{2}v$.
- Raider makes TIOLI offer b to Firm.
- PBE: Firm sells iff $v < b \implies$ average value $\frac{3}{2}\left(\frac{1}{2}b\right) = \frac{3}{4}b < b$
- Optimal (lemon-flavoured) bid: $b^* = 0$
- Experimental Evidence: Bazerman and Samuelson (1985):
  - 59% of subjects bid in $[0.5, 0.75]$, and 92% bid more than 0.
  - Subjects lose money on average.

Inference and Learning from Others

Cursed equilibrium:

- Firm accepts iff $v < b$.
- Raider understands higher $b \implies$ more likely accepted: . predicts true prob(b accepted) $= b$.
  - Doesn't attend to lemons problem when bidding?
- As if Raider thinks sales independent of v.
- Raider solves $\max_b b(\frac{3}{2}E[v] - b) = \frac{3}{4}b - b^2$ and $b^* = \frac{3}{8} = .38$.
- (Note: even most extreme form of our model too low.)

Inference and Learning from Others

Eyster, Rabin, and Vayanos (2013):

- Adapt CE to market model.
- Cursed traders don't think through information in market prices.
    - Don't ask: why is that person trading with me?
    - Retro! ... pre-Grossman REE revolution.
    - Old school not 100% wrong
- Simple, one-period asset market (with returns realized after trade)
- Variously all cursed vs. some rational, some cursed.

Inference and Learning from Others

CEE vs. "REE": $\rightleftharpoons$

- Many specific predictions (e.g., "momentum") $\rightleftharpoons$
- In REE: $\rightleftharpoons$
  - Traders might trade for liquidity, etc. $\rightleftharpoons$
  - But no "speculative trade" $\rightleftharpoons$
  - Not plausible volume. $\rightleftharpoons$
- Biggest and simplest implication of CEE: $\rightleftharpoons$
  - **High volume of trade.** $\rightleftharpoons$
- Rational traders: more private info $\implies$ less trade. $\rightleftharpoons$
  - Rationals become more suspicious of others' motivation to trade $\rightleftharpoons$
- With cursedness: more private info can imply *more* trade. $\rightleftharpoons$
  - Increasing difference in opinion, w/o understanding adverse selection.

Inference and Learning from Others

Alternative approaches?

- Overconfidence
  - People think own signals are better than they are.
  - (And maybe that others' signals worse)
- **Agreeing to disagree, non-common priors**
  - ATD directly, or to "close" overconfidence models.
  - Traders aware of disagreements.
  - Seek bets based on this!
  - (This explanation ... massive degrees of freedom)
  - And often not plausible:
- ERV show experimental evidence
  - Where overconfidence incoherent
  - Priors don't matter

Inference and Learning from Others

Return to Bazerman and Samuleson:

- Recall: uninformed Raider over-bids on perfectly informed Firm.
- Raider overbidding because of "overconfidence"?
  - Knows nothing, and surely knows she knows nothing.
- Raider agreeing to disagree with Firm's beliefs on v?
  - Surely believes Firm is exactly right.
  - Rather: she is not fully thinking through the lemons problem.

Repeated in much of other evidence:

- People behave in ways that look like market trading due to overconfidence and ATD.
- But in settings where neither could be going on.

Doesn't prove OC/ATD isn't going on in markets!

- But clarifies alternative, and suggests worry.

Inference and Learning from Others

ATD driving force of small investors trading against the market? $\rightleftharpoons$

- **Do they think they are outsmarting experts and insiders?**$\rightleftharpoons$
- **Or not attending that they are trading against them?**$\rightleftharpoons$
- May think they are aligned with "smart money" against fools?$\rightleftharpoons$

ERV shows that, even if true, overconfidence/ATD unlikely to explain high volume in large, "high-information" markets.$\rightleftharpoons$ Roughly:$\rightleftharpoons$

- If a trader believes $\rightleftharpoons$
    - lots of information besides her own insights,$\rightleftharpoons$
    - then just like REE logic $\Longrightarrow$ little speculative trade in large markets.$\rightleftharpoons$
    - Degree overconfidence needed w/o cursedness?

Inferential Naivety in Observational Learning $\rightleftharpoons$

- People may extract information (not fully cursed) from others. $\rightleftharpoons$
- But take this information "at face value" $\rightleftharpoons$
- Portable and pinned down universal definition $\rightleftharpoons$
  - But now talk solely about observational learning.

$\looparrowright$

Inference and Learning from Others

Rational-Herding Literature:$\rightleftharpoons$

- People infer from actions of those with similar tastes.$\rightleftharpoons$
- Rational imitation.$\rightleftharpoons$
- Herds may start & last on wrong choice.$\rightleftharpoons$
- All realize that others also imitating$\rightleftharpoons$
- Understand *inherent* redundancy in others' behavior$\rightleftharpoons$
- Don't imitate very much.

$\looparrowright$

Inference and Learning from Others

- We are skeptical people so reluctant to imitate.
- **And we should care a lot about this:**
  - Examples of theories that generate extensive imitation ... predict severe badnesses in societal beliefs.
- Extensive imitation $\implies$ not rational
- Extensive imitation $\implies$ social confirmation bias & false beliefs.

Now:

- **Behavioral implications** of full rationality in observational learning
- Very different implications of inferential naivety.

Review of Canonical Rational-Herding Models$\rightleftharpoons$

Banerjee (1992), Bikhchandani, Hirshleifer and Welch (1992).$\rightleftharpoons$

- Sequentially move, all privately informed, observe actions and order but not info of those before.$\rightleftharpoons$
- No direct externalities.$\rightleftharpoons$
- Canonical example:

$\looparrowright$

Inference and Learning from Others

Canonical model: Either A or B is good—but not both (?)—and binary private i.i.d. signals. A when you believe $\omega = 1$, B when $\omega = 0$. α signal of $\omega = 1$, β of $\omega = 0$;⇌

player	*signal*	*action*
1	α	A
2	β	B
3	α	A
4	α	A
5	β	$[A]$
6	α	$[A]$
7	β	$[A]$
8	α	$[A]$
9	β	$[A]$
10	β	$[A]$

↬

Inference and Learning from Others

Efficiency facts of rational herding models:

- Observing others always helps in expected terms.
- High likelihood wrong herds only if those herds are unconfident.
- Rational-herding literature is about failure to aggregate information, not of society (frequently) thinking it knows things it doesn't.
  - (Debated in literature: is even non-aggregation really likely?)

We claim: $\rightleftharpoons$

- Canonical example, connotation of literature misleading. $\rightleftharpoons$
- Limits to imitation far bigger punchline than the imitation itself. $\rightleftharpoons$
- We think the non-imitation is unrealistic. $\rightleftharpoons$
- And (later show) it matters. $\rightleftharpoons$

Modification of the standard two-restaurant model of social learning. $\rightleftharpoons$

- Two restaurants in town, $\rightleftharpoons$
  - A and B, $p(A\ good, B\ bad) = p(B\ good, A\ bad) = .5.$ $\rightleftharpoons$
  - Two states: $\omega_A \rightarrow A$ is good, $\omega_B \rightarrow B$ is good. $\rightleftharpoons$
  - Binary-state model universal. $\rightleftharpoons$ ... and weird.

$\looparrowright$

Inference and Learning from Others

- Each of ∞ diners receives private signals $\in \{\alpha, \beta, \emptyset\}$
- The signals are *i.i.d.* conditional on the state,
  - α supports ω_A,
  - β supports ω_B,
  - $\emptyset$ uninformative.
- For each Player k,
  - $\Pr[s_k = \alpha | \omega_A] = \Pr[s_k = \beta | \omega_B] = .7(1 - \eta)$ and
  - $\Pr[\emptyset | \omega_A] = \Pr[\emptyset | \omega_B] = \eta$.
  - $\eta = 0$, canonical binary-signal information structure.
  - When $\eta \to 1$, information is very rare.
  - (Lots results independent of η)

Inference and Learning from Others

- Each Player k chooses among nine choices:
  - dine in Restaurant A, dine in Restaurant B, or dine at home.
  - Goes to a restaurant if she thinks there is more than 60% chance it is good, and stays at home if that is not true at either restaurant.
- Depending on confidence in restaurant's quality, may go alone, or take one, two, or three of her relatives.
- Superscripts for the number of people she takes:

$p(\omega_A)$	[0,10),[10,20),[20,30),[30,40)	[40,60]	(60,70],(70,80],(80,90],(90,100]
Choice	$B^{+++}, B^{++}, B^{+}, B$	H	$A, A^{+}, A^{++}, A^{+++}$

Three people choose restaurants each period,⇌

- Signal conditionally i.i.d. given state⇌
- Each doing so after observing her own signal,⇌ and the full actions (three locations, and party size), in order,⇌ taken in all previous periods.

This example is clearly very contrived.⇌ But scout's honor ... ⇌

- General punchlines not based on specifics.

↪

What predictions does full rationality make?

- $\emptyset$ signal, observes nothing but H $\longrightarrow$ stay home.
- α or β signal, observes nothing but H $\longrightarrow$ go to restaurant.
- (alone, because beliefs exactly .7 $\longrightarrow$ alone).

Suppose in period 2 observe that exactly one person has gone to Restaurant A in period 1.

- What do as a function of your signal?
- You will realize that the three signals in period 1 were $\{\alpha, \emptyset, \emptyset\}$.
  - $\beta \longrightarrow H$.
  - $\emptyset \longrightarrow A$.
  - $\alpha \longrightarrow A^{++}$

Inference and Learning from Others

If observe: $\rightleftharpoons$

	actions
Period 1:	$\{A, H, H\}$ $\rightleftharpoons$
Period 2:	$\{A, A, A\}$

What do (as function of signal)?
$\looparrowright$

Inference and Learning from Others

	actions	response	signals
Period 1:	$\{A, H, H\}$		$\{\alpha, \emptyset, \emptyset\}$
Period 2:	$\{A, A, A\}$		$\{\emptyset, \emptyset, \emptyset\}$
Period 3:		$\beta \to H, \emptyset \to A, \alpha \to A^{++}$	

- Key logic: guys in period 2 did *not* get any additional information.
    - (If did, would not have gone alone.)
    - Period 3: rationally realize no new information in Period-2 followers.

Inference and Learning from Others

If observe: $\rightleftharpoons$

	actions	
Period 1:	$\{A, H, H\}$	
Period 2:	$\{A, A, A\}$	$\rightleftharpoons$
Period 3:	$\{A, A, A\}$	
Period 4:	$\{A, A, A\}$	
Period 5:	$\{A, A, A\}$	

What do as (as function of signal)?

$\looparrowright$

Inference and Learning from Others

	actions	response		signals
Period 1:	$\{A, H, H\}$			$\{\alpha, \emptyset, \emptyset\}$
Period 2:	$\{A, A, A\}$			$\{\emptyset, \emptyset, \emptyset\}$
Period 3:	$\{A, A, A\}$		$\rightleftharpoons$	$\{\emptyset, \emptyset, \emptyset\}$
Period 4:	$\{A, A, A\}$			$\{\emptyset, \emptyset, \emptyset\}$
Period 5:	$\{A, A, A\}$			$\{\emptyset, \emptyset, \emptyset\}$

Period 6: $\beta \to H, \ \emptyset \to A, \ \alpha \to A^{++}$

- Understanding redundancy information in actions: hard.$_{\rightleftharpoons}$
- But it matters a lot.

$\looparrowright$

Inference and Learning from Others

If observe: $\rightleftharpoons$

	actions
Period 1:	$\{A, H, H\}$ $\rightleftharpoons$
Period 2:	$\{A, A, H\}$

What do (as function of signal)?

$\looparrowright$

Herding without sufficiently increased enthusiasm is a bad sign:

	actions	response	signals
Period 1:	$\{A, H, H\}$		$\{\alpha, \emptyset, \emptyset\}$
Period 2:	$\{A, A, H\}$		$\rightleftharpoons$ $\{\emptyset, \emptyset, \beta\}$
Period 3:		$\beta \longrightarrow B$, $\emptyset \longrightarrow H$, $\alpha \longrightarrow A$	

$\rightleftharpoons$

3 A, 3 H $\longrightarrow \omega_A, \omega_B$ equally likely!$\rightleftharpoons$

- Do we get that?

$\looparrowright$

Inference and Learning from Others

If observe: $\rightleftharpoons$

	actions
Period 1:	$\{A, H, H\}$ $\rightleftharpoons$
Period 2:	$\{A, H, H\}$

What do (as function of signal)?

$\looparrowright$

Inference and Learning from Others

	actions	response	signals	
Period 1:	$\{A, H, H\}$		$\{\alpha, \emptyset, \emptyset\}$	
Period 2:	$\{A, H, H\}$	$\rightleftharpoons$	$\{\emptyset, \beta, \beta\}$	$\rightleftharpoons$
Period 3:		$\beta \longrightarrow B^{++}$, $\emptyset \longrightarrow B$, $\alpha \longrightarrow H$		

You shouldn't go to A even if get α!

$\looparrowright$

If observe: $\rightleftharpoons$

	actions
Period 1:	$\{A, H, H\}$ $\rightleftharpoons$
Period 2:	$\{H, H, H\}$

What do (as function of signal)?
$\looparrowright$

Inference and Learning from Others

	actions	response		signals
Period 1:	$\{A, H, H\}$			$\{\alpha, \emptyset, \emptyset\}$
Period 2:	$\{H, H, H\}$		$\rightleftharpoons$	$\{\beta, \beta, \beta\}$
Period 3:		$\beta \longrightarrow B^{+++}, \emptyset \longrightarrow B^{++}, \alpha \longrightarrow B$		

Go to B no matter what!

$\looparrowright$

Inference and Learning from Others

If observe: $\rightleftharpoons$

	actions	
Period 1:	$\{A, H, H\}$	$\rightleftharpoons$
Period 2:	$\{A^{++}, A, A\}$	
Period 3:	$\{A^{++}, A, A\}$	

What do (as function of signal)?

$\looparrowright$

	actions	response		signals
Period 1:	$\{A, H, H\}$			$\{\alpha, \emptyset, \emptyset\}$
Period 2:	$\{A^{++}, A, A\}$		$\rightleftharpoons$	$\{\alpha, \emptyset, \emptyset\}$
Period 3:	$\{A^{++}, A, A\}$			$\{\emptyset, \beta, \beta\}$
Period 4:		$\beta \to B,\ \emptyset \to H,\ \alpha \to A$		

$\looparrowright$

Inference and Learning from Others

If observe:$\rightleftharpoons$

	actions
Period 1:	$\{A, H, H\}$
Period 2:	$\{A^{++}, A, A\}$
Period 3:	$\{A^{++}, A^{++}, A\}$
Period 4:	$\{A^{++}, A^{++}, A\}$
Period 5:	$\{A^{++}, A^{++}, A^{++}\}$

$\rightleftharpoons$

What do (as function of signal)?

$\looparrowright$

Inference and Learning from Others

	actions	response	signals
Period 1:	$\{A, H, H\}$		$\{\alpha, \emptyset, \emptyset$
Period 2:	$\{A^{++}, A, A\}$		$\{\alpha, \emptyset, \emptyset$
Period 3:	$\{A^{++}, A^{++}, A\}$		$\rightleftharpoons$ $\{\emptyset, \emptyset, \beta$
Period 4:	$\{A^{++}, A^{++}, A\}$		$\{\alpha, \alpha, \emptyset$
Period 5:	$\{A^{++}, A^{++}, A^{++}\}$		$\{\beta, \beta, \beta$
Period 6:		$\beta \rightarrow H$, $\emptyset \rightarrow A$, $\alpha \rightarrow A^{++}$	

Will a β signal help stop the herd?

$\looparrowright$

Inference and Learning from Others

If observe: $\rightleftharpoons$

	actions	
Period 1:	$\{A, A, A\}$	$\rightleftharpoons$
Period 2:	$\{A^{++}, A^{++}, A^{++}\}$	

What do (as function of signal)?

$\looparrowright$

Inference and Learning from Others

	actions	response	signals
Period 1:	$\{A, A, A\}$		$\{\alpha, \alpha, \alpha\}$
Period 2:	$\{A^{++}, A^{++}, A^{++}\}$		$\{\beta, \beta, \beta\}$
Period 3:		$\beta \to B,\ \emptyset \to H,\ \alpha \to A$	

- Enough.
- Things far more complicated if don't observe order, don't observe all, or heterogenous preferences
  - But nothing makes the severe limits to imitation go away
- **Others' beliefs massively correlated**
  - $\Longrightarrow$ **musn't imitate too much.**

Inference and Learning from Others

Same setting (same signals, players per period, etc.) but:⇌

- Cannot observe order of play.⇌
- **Signals rare**⇌
- In period 3, if see ⇌
  - If see $\{H, H, H, H, H, H\}$, then believe ⇌ .5⇌
  - If see $\{A, H, H, H, H, H\}$, then believe ⇌ .7⇌
  - If see $\{A, A, H, H, H, H\}$, then believe ⇌ .84⇌
  - If see $\{A, A, A, H, H, H\}$, then believe ⇌ .5⇌
  - If see $\{A, A, A, A, H, H\}$, then believe ⇌ .7⇌
  - If see $\{A, A, A, A, A, H\}$, then believe ⇌ .3⇌

One old and one new example:

Inference and Learning from Others

	actions	response		signals
Period 1:	$\{A, H, H\}$		$\rightleftharpoons$	$\{\alpha, \emptyset, \emptyset\}$
Period 2:	$\{H, H, H\}$			$\{\beta, \beta, \beta\}$
Period 3:		$\beta \to B^{+++}, \emptyset \to B^{++}, \alpha \to B$		

	actions	response		signals
Period 1:	$\{B, H, H\}$		$\rightleftharpoons$	$\{\beta, \emptyset, \emptyset\}$
Period 2:	$\{H, H, H\}$			$\{\alpha, \alpha, \alpha\}$
Period 3:		$\beta \to A, \emptyset \to A^{++}, \alpha \to A^{+++}$		

Anti-imitation!

Inference and Learning from Others

Lesson?$\rightleftharpoons$

- Choosing a restaurant may be hard!$\rightleftharpoons$

More interesting lesson:$\rightleftharpoons$

- Most natural learning environments...$\rightleftharpoons$
  - Great deal redundancy.$\rightleftharpoons$
- Theorem 1$\rightleftharpoons$ ('False but not misleading'):$\rightleftharpoons$
  - Almost all non-single-file environments, rationality implies 'anti-imitation'.$\rightleftharpoons$
  - Intuition: if two recent guys both imitating earlier guy but not each other—imitate both, $\rightleftharpoons$
  - **but subtract off source of correlation... earlier guy**.$\rightleftharpoons$
- Theorem 2:$\rightleftharpoons$
  - What few environments that don't demand 'anti-imitation' $\rightleftharpoons$ (e.g., single-file),$\rightleftharpoons$ imitate all you see as if seeing only one person.

Harder to see: $\rightleftharpoons$

- Rational observational learning in this case: $\rightleftharpoons$
  - Eventually will herd on $\{B^{+++}\}$ or $\{A^{+++}\}$. $\rightleftharpoons$
  - More than 95% of time $\rightarrow$ right restaurant. $\rightleftharpoons$
  - Intuition: any lesser certainty, contrary signal will moderate behavior. $\rightleftharpoons$
- **When signals rare:** $\rightleftharpoons$
  - Roughly 30% of time herd starts in wrong direction, $\rightleftharpoons$
  - stays wrong $< 5\%$ time. $\rightleftharpoons$
  - $\rightarrow > 25\%$ of time: herd in wrong direction followed by reversal... $\rightleftharpoons$
  - somebody observing at least 50 people going to one restaurant and none to other decides stay home based on opposite signal.

$\looparrowright$

Inference and Learning from Others

Rationality does not just demand great care.

- demands something people get wrong in systematic direction.

What happens if instead people neglect redundancy?

- ER (2010) particular extreme form.
- "Britney" behaves as if all she is observing are independent.

When signals are rare,

- Tommy:
  - less than 5% chance society converges to wrong restaurant,
  - Everybody less than 96% sure they have right one.
- Britney:
  - ∼30% chance convergence to wrong restaurant
  - Everybody around ∼100% sure they are right.

Inference and Learning from Others

Eyster-Rabin (2014) ... ⇌

- In many environments, tiny amounts of naive redundancy neglect can lead society astray.⇌
- In example:⇌ For weak signals, **any** learning rule in which⇌
    - everybody (?)⇌ who sees 50 or more go one restaurant and none go to other follows the herd no matter signal⇌
    - will lead to 30% chance of false herd.

↬