Present Bias Amplifies the Household Balance-Sheet Channels of Macroeconomic Policy*

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Abstract

We study the effect of monetary and fiscal policy in a heterogeneous-agent model where households have present-biased time preferences and naive beliefs. The model features a liquid asset and illiquid home equity, which households can use as collateral for borrowing. Because present bias substantially increases households' marginal propensity to consume (MPC), present bias increases the impact of fiscal policy. Present bias also amplifies the effect of monetary policy but, at the same time, slows down the speed of monetary transmission. Interest rate cuts incentivize households to conduct cash-out refinances, which become targeted liquidity-injections to high-MPC households. But present bias also introduces a motive for households to procrastinate refinancing their mortgages, which slows down the speed with which this monetary channel operates.

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1 Introduction

The idea that dynamically inconsistent preferences may alter individuals' dynamic choices has a long tradition going back to seminal work by Strotz (1956). A particular form of dynamic inconsistency, present bias, has received empirical support in both laboratory and field studies (e.g., Ashraf et al., 2006; Augenblick et al., 2015; Laibson et al., 2020; and the review by Cohen et al., 2020). Present bias implies that the current self draws a sharp distinction between a util that is experienced now versus a util experienced one time unit in the future, but draws relatively little distinction between a util consumed at any other two successive future dates (Phelps and Pollak, 1968; Laibson, 1997).[1]

The two most commonly used tools of macroeconomic stabilization policy – monetary and fiscal policy – operate in large part by affecting household consumption and investment decisions, two leading examples of the types of dynamic choices that are affected by present bias. It is therefore natural to ask whether and to what extent present bias alters the potency of these policy tools. To answer these questions, we develop and calibrate a heterogeneous-agent consumption model and use the model to evaluate the impact of present bias on policy outcomes.

Our modeling framework is motivated in part by Campbell's (2006) concept of *positive household finance*: households face a complex financial planning problem, and household behavior is influenced by a range of psychological factors. Our model aims to capture the complexities of household balance sheets that are important for the transmission of monetary and fiscal policy, as well as the channels through which present bias interacts with these balance sheet features.

We set our model in partial equilibrium in order to focus on the details of the household problem. Time is continuous, and we compare the exponential-discounting benchmark to a tractable, and empirically realistic, continuous-time limit of present-biased discounting. In addition to present bias, we assume that households have naive beliefs, meaning that households do not foresee their own future present bias (Strotz, 1956; Akerlof, 1991; O'Donoghue and Rabin, 1999). The modeling of present bias in continuous time builds on the foundational work of Barro (1999) and Luttmer and Mariotti (2003), and our specific approach follows Harris and Laibson (2013).[2]

The household budget constraint includes stochastic auto-correlated labor income and stochastic auto-correlated interest rates. On the asset side of the household balance sheet, the model features a liquid savings account and an illiquid home. Households can build

[1] Throughout this paper, we use "present bias" to refer specifically to quasi-hyperbolic discounting. Other equivalent terms are: "β-δ preferences" in discrete-time models; and "Instantaneous Gratification preferences" in continuous-time models like the one in this paper.

[2] See Laibson and Maxted (2020) and Maxted (2020) for related extensions of Harris and Laibson (2013).

a buffer stock of liquid wealth to insure against income fluctuations, and can accumulate home equity by paying down their mortgage. On the liabilities side of the balance sheet, households have access to two forms of debt: credit cards and mortgages. Households can borrow on their credit cards up to a calibrated limit. If they have enough home equity, they can also borrow against their home by refinancing their mortgage. We calibrate our heterogeneous-agent model to reproduce two empirical regularities on household balance sheets: the average quantity of credit card debt and the average loan-to-value ratio in the housing market.[3]

In order to focus on the effects of mortgage refinancing, we study homeowners, a large fraction of the population: two-thirds of U.S. housing units are owner-occupied. Homeowners represent an even larger fraction of aggregate income and consumption, and homeowners are likely the most important channels for fiscal (Cloyne and Surico, 2017) and monetary policy (Wong, 2019; Cloyne et al., 2020).

Our main result is that relative to exponential discounting, present bias amplifies the balance-sheet channels of both fiscal and monetary policy, but with some important added subtlety in the case of monetary policy due to refinancing procrastination.

Fiscal policy is powerfully enhanced by present bias, because present bias sharply raises households' average marginal propensity to consume (MPC) (Angeletos et al., 2001). Present bias increases both the share of liquidity constrained households and the MPC at the constraint. With present bias, the liquidity of fiscal transfers becomes vital. Because the consumption response is driven by constrained households, it is critical that fiscal policy comes as a liquid transfer (e.g., checks in the mail) rather than an illiquid transfer (e.g., mortgage principal reductions).

We show that the effects of present bias are quantitatively large. In our Exponential Benchmark model the quarterly MPC is predicted to be 4% and the quarterly marginal propensity for expenditure (MPX), which includes spending on both nondurables and durables, is predicted to be 14%. In our Present-Bias Benchmark, the MPC rises from 4% to 13% and the MPX rises from 14% to 32%. These higher propensities to consume and spend are more consistent with the empirical literature: estimates of quarterly MPCs for nondurables are on the order of 15-25%, and estimates of quarterly MPXs are often two- to three-times larger.[4]

[3] The version of our model with exponential discounting is similar to that of Guerrieri et al. (2020) with two main differences: we assume that housing is fixed and cannot be adjusted while they model a costly housing adjustment decision, and our model features credit card debt while theirs does not. Also see Berger et al. (2018a), Berger et al. (2018b), Eichenbaum et al. (2018), Wong (2019), Kaplan et al. (2020), and Kinnerud (2021) for related models of housing and mortgage refinancing decisions. Like us, McKay and Wieland (2019) and Guerrieri et al. (2020) use the continuous-time methods of Achdou et al. (2021) to solve their models.

[4] For empirical MPC estimates, see for example Johnson et al. (2006), Parker et al. (2013), and the discussion of this literature in Kaplan et al. (2018) and Kaplan and Violante (2014). For empirical MPX

Present bias also amplifies the overall effect of monetary policy, but slows down the speed of monetary transmission (an offsetting effect). Interest rate cuts incentivize households to conduct cash-out refinances, which serve as targeted liquidity-injections to households with especially high MPCs (because they are near their liquidity constraint). But present bias with naive beliefs also introduces a motivation for households to procrastinate on refinancing their mortgage, which substantially slows down the speed at which this channel operates. Naive present bias implies that households will delay completing immediate-cost, delayed-reward tasks such as mortgage refinancing, which tends to take weeks and requires the borrower to go through the effortful process of negotiating with lenders, gathering documents, and filling out paperwork. Naive households will keep delaying refinancing, all the while (counterfactually) believing that the task will get done in the near future.

A noteworthy feature of our model is that present bias amplifies the direct effect of monetary policy on household consumption while, at the same time, also delivering larger MPCs. This is in contrast to standard heterogeneous-agent models, where modeling choices that amplify MPCs typically deliver *smaller* consumption responses to interest rate changes (Werning, 2015; Olivi, 2017; Kaplan et al., 2018; Auclert, 2019; Slacalek et al., 2020).[5] Our model instead delivers a larger responsiveness to monetary policy precisely *because* of the higher MPCs: interest rate cuts incentivize households to conduct cash-out refinances, which become targeted liquidity-injections to high-MPC households.

Our final result is that present bias reshapes the distributional consequences of macroeconomic stabilization policy. Without present bias, the refinancing channel of monetary policy is a powerful means of increasing the consumption of low-consumption households. With present bias, this channel is weakened because households procrastinate on refinancing. Instead, fiscal policy becomes a more effective policy tool for increasing the consumption of low-consumption households. Present bias also heightens the importance of understanding the distributional consequences of stabilization policy, because the present-biased economy features a larger mass of low-consumption households.

Though our model is stylized, the steady state of the present-biased economy replicates a variety of empirical patterns from the household finance literature that have, collectively, proven difficult to replicate in models with exponential discounting. The present-biased economy generates empirically-plausible levels of high-cost credit card borrowing by homeowners (Zinman, 2015), cash-out behavior, and loan-to-value (LTV) ratios. It also features

estimates, see for example Parker et al. (2013), Fagereng et al. (2019a), Di Maggio et al. (2020), and the discussion in Laibson et al. (2021).

[5] This statement is made precise by Auclert (2019) and Olivi (2017) who show that, in a standard one-asset consumption-saving problem, a household's MPC is a "sufficient statistic" to determine both income and substitution effects of interest rate changes and, in particular, enters the substitution effect with a negative sign. For example, Auclert (2019) shows that the substitution effect of a small, one-period interest rate change dr on consumption c is $d \log c^{\text{IES}} = \text{IES} \times (1 - \text{MPC}) \times dr$, where IES is the intertemporal elasticity of substitution — also see the exposition in Slacalek et al. (2020).

a buildup of liquidity-constrained households that is consistent with empirical estimates of households' propensity to spend out of increases in credit card borrowing limits (Gross and Souleles, 2002; Agarwal et al., 2018). Present-biased households struggle to smooth consumption, resulting in a consumption function with discontinuities at the borrowing constraint (Ganong and Noel, 2019). Present bias delivers larger MPCs and MPXs, as well as MPCs and MPXs that remain elevated for large shocks (Fagereng et al., 2019a).[6] The time-profile of consumer spending is consistent with the intertemporal MPC evidence presented in Auclert et al. (2018) (using data from Fagereng et al. (2019a)). Moreover, the present-biased economy generates differential MPCs out of liquid cash transfers versus illiquid home equity increases, a pattern shown empirically by Ganong and Noel (2020).

Finally, there is a large literature documenting refinancing inertia: the proclivity for households to delay refinancing when it is financially optimal to do so (e.g., Keys et al., 2016; Johnson et al., 2019; Andersen et al., 2020). We show that present bias with naive beliefs provides a natural motivation for this behavior.

Section 2 lays out our model of the household balance sheet mechanisms of monetary and fiscal policy. Section 3 characterizes the effect of present bias on household consumption and refinancing decisions. Section 4 discusses our calibration and what it implies for the model's steady state. Section 5 presents our main results about macroeconomic stabilization policy. Section 6 examines the robustness of our results to aggregate house price and income shocks. Section 7 extends our monetary policy analysis to an economy with adjustable-rate mortgages, and discusses general equilibrium effects. Section 8 concludes.

2 A Model of Household Finances with Present Bias

Our model is set in partial equilibrium. The goal of the model is to capture the household balance-sheet channels through which present bias impacts fiscal and monetary policy. Abstracting from general equilibrium considerations simplifies the analysis and allows for a richer investigation of the institutional factors that affect the household problem. Our partial equilibrium results should be interpreted as just one part of the overall macroeconomic analysis, providing inputs for a full general equilibrium analysis. We return to this theme in Section 7.2, which briefly discusses how present bias may alter the transmission of macroeconomic policy in general equilibrium.

[6] Besides present bias, other deviations from standard exponential discounting also have the potential to help match empirical MPCs. For example, Attanasio et al. (2020) study a model with both liquid and illiquid assets and show that temptation preferences can help in this regard. Lian (2021) presents a general framework showing that the anticipation of future mistakes increases MPCs.

2.1 The Household Balance Sheet

There is a unit mass of households that are heterogeneous in their wealth and income. Here we outline the evolution of each household's balance sheet.

Budget Constraints. Each household faces idiosyncratic income risk. The household's income is denoted y_t, and y_t follows a finite state Poisson process. We normalize the average income flow to 1.

Households hold three types of assets: liquid wealth b_t, illiquid housing h, and a fixed-rate mortgage m_t. For simplicity we assume that each household is endowed with a home of fixed value h.[7] The remainder of the household balance sheet evolves as follows:

$$\dot{b}_t = y_t + r_t b_t + \omega^{cc} b_t^- - (r_t^m + \xi) m_t - c_t, \tag{1}$$

$$\dot{m}_t = -\xi m_t, \tag{2}$$

subject to the borrowing constraint $b_t \geq \underline{b}$ and the loan-to-value (LTV) constraint $m_t \in [0, \theta h]$. Equation (1) characterizes the evolution of liquid wealth b_t. Equation (2) describes the evolution of mortgage balances. Explaining equation (1), households earn income y_t and have a consumption outflow of c_t. The return to liquid wealth is given by $r_t b_t + \omega^{cc} b_t^-$, where $b_t^- = \min\{b_t, 0\}$. $\omega^{cc} > 0$ is a credit card borrowing wedge, which generates a "soft constraint" at $b = 0$ (Kaplan and Violante, 2014; Achdou et al., 2021). The household's total mortgage payment is captured by $(r_t^m + \xi) m_t$. This is composed of a mortgage interest payment ($r_t^m \times m_t$) and a principal repayment ($\xi \times m_t$). To economize on state variables, we make the slightly non-standard assumption that the household pays down its mortgage at a constant proportional rate ξ (Agarwal et al., 2013). The more realistic assumption of a constant flow payment would require an additional state variable.

The borrowing constraint is important for our results, so we emphasize its effects here. In continuous time, the consumption rate c_t is unconstrained for all $b_t > \underline{b}$: any finite rate of consumption can be adopted without violating the borrowing constraint, so long as that rate of consumption persists for a short enough period of time (Achdou et al., 2021). However, at the liquid-wealth constraint of $b_t = \underline{b}$ the household is restricted to consume at a rate

$$c_t \leq y_t + (r_t + \omega^{cc})\underline{b} - (r_t^m + \xi) m_t. \tag{3}$$

In short, consumption will be unconstrained for all $b_t > \underline{b}$, but features an occasionally-binding constraint when $b_t = \underline{b}$.

[7] We study only households' short-run response to fiscal and monetary policy, and house prices are slow-moving over short horizons (Case and Shiller, 1989). Section 6.1 presents an extension with house price shocks.

Equations (1) and (2) characterize how the household's balance sheet evolves continuously. Households also have the option of paying a fixed cost to discretely adjust their mortgage. We provide details of this discrete adjustment process, which includes the option to refinance, further below after outlining the interest rate process.

Interest Rates. The real interest rate is denoted r_t. We assume that r_t follows a finite state Poisson process. Since our goal is to study the effect of changes in mortgage rates we will treat r_t as a long rate (e.g., 10-year TIPS). When discussing monetary policy in Section 5, we implicitly assume that the Federal Reserve is implementing the necessary short rate adjustments to generate the corresponding changes in long rate r_t. The calibration of the interest rate process is discussed in Section 4.1. Households have rational expectations about this interest rate process. Because this paper studies the refinancing channel of monetary policy, it is important that households have reasonable expectations about how mortgage rates evolve over time.[8]

Each household pays a mortgage interest rate r_t^m. To capture features of the U.S. mortgage market, this mortgage rate is fixed until the household decides to refinance. At the time of refinancing, the household switches to a new mortgage rate of $r_t^m = r_t + \omega^m$, where ω^m represents the mortgage borrowing wedge over the current interest rate r_t.

Discrete Adjustments and Mortgage Refinancing. The laws of motion specified by equations (1) and (2) describe how liquid wealth and mortgage balances evolve continuously. In the model, households also have two methods for discretely adjusting their balance sheet position between liquid wealth and illiquid home equity. First, households can pay a small fixed cost to *prepay* their mortgage. Second, households can pay a larger fixed cost to *refinance* their mortgage. We provide details of these options below.

A household's first adjustment option is to prepay its mortgage. Prepayment requires a small fixed cost of κ^{prepay}.[9] We introduce this prepayment option because mortgage contracts typically allow households the option to pay down their mortgage faster than contractually required. In particular, the household chooses a new liquid wealth value b' and a new mortgage value m' such that

$$b' - m' = b_t - m_t - \kappa^{prepay}, \text{ subject to } m' \in [0, m_t) \text{ and } b' \geq \underline{b}. \tag{4}$$

Prepayment does not affect the mortgage interest rate, which remains the same as before the adjustment decision. By using part of their liquid wealth to prepay their mortgage,

[8]Importantly, this means that interest rate shocks in our model are not "MIT shocks." This feature differentiates our model from many other heterogeneous-agent models that study monetary policy.

[9]We impose a small fixed cost for numerical stability. This small fixed cost can be thought of as capturing, for example, the time cost of the additional budgeting required to make a mortgage prepayment.

households are effectively shifting their portfolio from a low-return liquid asset to a high-return illiquid asset.[10] Home equity is illiquid because it can only be accessed through a cash-out refinance. But, the benefit of accumulating home equity is that wedge ω^m makes mortgage debt costly. Households will therefore first build a buffer-stock of liquid wealth and then use additional liquidity to prepay their mortgage.

A household's second adjustment option is to refinance its mortgage. Refinancing requires payment of a fixed cost of κ^{refi}. When refinancing, the household chooses a new liquid wealth value b' and a new mortgage value m' such that

$$b' - m' = b_t - m_t - \kappa^{refi}, \text{ subject to } m' \in [0, \theta h] \text{ and } b' \geq \underline{b}. \tag{5}$$

By refinancing, the household also resets the interest rate on its mortgage to $r_t^m = r_t + \omega^m$.

Though refinancing requires up-front costs, there are two reasons why households may choose to refinance. First, if the market interest rate falls then refinancing "locks in" lower mortgage interest payments. Second, refinancing allows households to rebalance their asset allocation across liquid wealth and illiquid home equity. For example, a cash-out refinance lets households convert illiquid home equity into liquid wealth. Accessing home equity is useful during spells of low income (consumption smoothing), and also as a means of converting costly credit card debt into cheaper mortgage debt.[11]

These two motives for refinancing are not mutually exclusive. Consider an interest rate cut. Once a household decides to pay the refinancing fixed cost of κ^{refi}, the case where it is optimal to not rebalance (i.e., $m' = m_t$) is a knife-edge case. Indeed, one of our main results is that interest rate cuts can be highly stimulative precisely because they induce a wave of home-equity extractions.

We assume that both types of discrete adjustments require a small effort cost $\bar{\varepsilon}$ (in addition to the monetary costs κ^{refi} and κ^{prepay}). This cost $\bar{\varepsilon}$ is intended to capture the effort associated with filling out paperwork, making budgeting decisions, negotiating with mortgage brokers, etc. We will show in Section 2.3 that this effort cost provides a natural mechanism for producing refinancing procrastination. In that section we also slightly generalize the setup by making the effort cost stochastic so as to capture the idea that households face occasional windows of time in which the marginal cost of making effortful budgeting adjustments is lower than normal, such as a free weekend or a cancelled afternoon meeting.

Finally, we note that our model does not allow for home equity lines of credit (HELOCs),

[10] Kaplan and Violante (2014) highlight how high-return illiquid assets can prevent households from building sizable liquid buffer stocks. Laibson et al. (2020) demonstrate that asset illiquidity combined with present bias allows lifecycle consumption-saving models to match the joint accumulation of credit card debt and illiquid savings that characterizes the balance sheets of many American households.

[11] There is a large literature on the consumption smoothing benefits of cash-out refinancing (e.g., Hurst and Stafford, 2004; Chen et al., 2020).

second mortgages, or reverse mortgages. These alternate mortgage products are much more likely to be used when interest rates are rising, in order to extract home equity without resetting the entire mortgage balance to a higher interest rate (Bhutta and Keys, 2016). We abstract from these alternate products because our paper focuses on the stimulative effect of interest rate cuts.

Other Structural Assumptions. To capture exogenous mortgage adjustment dynamics such as moving for a new job, we introduce an exogenous hazard rate λ^F at which households are forced to adjust their mortgage (and pay the cost to either refinance or prepay). We assume that households adjust their mortgage optimally when they are forced to do so.

To capture lifecycle dynamics, we assume that households retire at rate λ^R and are replaced by "first-time homeowners." To avoid needing an additional state variable we model retirement using a "perpetual youth" framework (Blanchard, 1985). A household who retires at time t receives a constant consumption flow of $y^R + \bar{r}(h - m_t + b_t)$ in perpetuity, where y^R is a fixed retirement income flow and $\bar{r}$ is the average interest rate. We denote the exponentially discounted value of the retirement consumption flow by $v^R(b_t, m_t) = [u(y^R + \bar{r}(h - m_t + b_t))]/\rho$, where ρ is an exponential discount rate. This parameterization captures a retirement pension of size y^R plus the annuity value of a household's assets at retirement. Retired households are dropped from the model and are replaced by new households who are endowed with the maximum LTV ratio of $m_t/h = \theta$.

Summary. The goal of our model is to provide a simple characterization of the household balance sheet features that are important for the conduct of macroeconomic stabilization policy. Our partial equilibrium model has five state variables: (b, m, y, r^m, r). Liquid wealth b and stochastic income y introduce uninsurable income risk and wealth heterogeneity. Mortgage m introduces a realistic role for housing, which is the primary illiquid asset held by most American households (Campbell, 2006). Time-varying interest rate r provides a role for monetary policy. Mortgage interest rate r^m introduces a refinancing motive, and allows us to study the refinancing channel of monetary policy.

To simplify notation, we will let $x = (b, m, y, r^m, r)$ denote the vector of state variables that characterize the household problem. We will use $g_t(x)$ to denote the distribution of households over the state space at time t. At any point in time, households can be heterogeneous in dimensions b, m, y and r^m. All households face the same time-varying market interest rate r_t.

2.2 Utility and Value

Utility. Households have CRRA utility over consumption:[12]

$$u(c) = \frac{c^{1-\gamma} - 1}{1-\gamma}.$$

Time Preferences: Instantaneous Gratification. This paper's key departure from rationality is the household's discount function. Households have naive *Instantaneous Gratification* (IG) time preferences. IG time preferences were first derived by Harris and Laibson (2013), and are extended in Laibson and Maxted (2020) and Maxted (2020).

In discrete time the quasi-hyperbolic discount function is given by $1, \beta\delta, \beta\delta^2, \beta\delta^3, \ldots$. IG preferences are the continuous-time limit of this discount function, where each self lives for a vanishingly small length of time.[13] For $t \geq 0$, the limiting IG discount function $D(t)$ is:

$$D(t) = \begin{cases} 1 & \text{if } t = 0 \\ \beta e^{-\rho t} & \text{if } t > 0 \end{cases}. \tag{6}$$

Since the current instantaneous self discounts all future selves by factor β, discount function $D(t)$ features a discontinuity at $t = 0$ whenever $\beta < 1$. The IG household values *instantaneous* utility flows, and all later utility is discounted by β. Note that $\beta = 1$ recovers the standard, time-consistent, exponential discount function.

We assume that households are *naive* about their present bias. This means that the current self is unaware of the self-control problems of future selves. Under naive present bias the current self discounts the utility flows of all future selves by $\beta < 1$, while expecting that all future selves will be exponential discounters ($\beta = 1$). The alternative would have been to assume that households are fully or partially sophisticated about their present bias. We assume naiveté for two reasons. First, it is theoretically and computationally easier to handle even though the effect on household decisions is likely similar.[14] Second, as we show below, (partial) naiveté can lead to high levels of procrastination.

As detailed in Laibson and Maxted (2020), IG preferences are a mathematically tractable limit case. They are not a psychologically realistic model of time preferences. The temporal

[12]We could also let households earn utility from housing h. We ignore this element since housing h is constant. This assumption is isomorphic to households having separable utility over consumption and housing, or CES utility with a unitary elasticity of substitution.

[13]Appendix B provides a heuristic argument for taking this limit, and Harris and Laibson (2013) provide a rigorous derivation.

[14]This assessment is based on Maxted (2020) who shows that naifs are observationally equivalent to sophisticates in a two-asset model similar to the one we study in this paper (though without effort costs and hence procrastination). For another elegant analysis of the more complicated case with sophistication, see Cao and Werning (2016).

division between "now" and "later" is certainly longer than a single instant dt.[15] Nonetheless, Laibson and Maxted (2020) show that discrete-time models with psychologically appropriate time-steps (e.g., each period lasts for one day or one week) are closely approximated by the continuous-time IG model. The current paper leverages the tractability of the IG approximation to study the effect of present bias on macroeconomic stabilization policy.

Remark 1 *IG time preferences are a generalization of standard time-consistent preferences. Exponential discounting is recovered by setting $\beta = 1$.*

Value Function ($\beta = 1$). We start by presenting the value function for an exponential ($\beta = 1$) household. We present the value function in steps, suppressing notation at first in order to clarify the structure of the household's decision-making problem. As a first step, the value function of a $\beta = 1$ household can be defined as the solution to the sequence problem:

$$
\begin{aligned}
v(x_0) &= \max_{\{c_t\},\tau} \mathbb{E}_0\left[\int_0^\tau e^{-\rho t}u(c_t)dt + e^{-\rho\tau}(v^*(x_\tau) - \bar{\varepsilon})\right] \quad \text{s.t. (1) and (2) hold and with} \\
v^*(x) &= \max\left\{v^{prepay}(x), v^{refi}(x)\right\} \\
v^{prepay}(x) &= \max_{b',m'} \; v(b', m', y, r^m, r) \qquad \text{s.t. prepayment constraint (4) holds} \\
v^{refi}(x) &= \max_{b',m'} \; v(b', m', y, r + \omega^m, r) \qquad \text{s.t. refinancing constraint (5) holds}
\end{aligned}
\tag{7}
$$

Equation (7) subsumes all Poisson shocks inside the expectation operator.

The integral $\mathbb{E}_0\left[\int_0^\tau e^{-\rho t}u(c_t)dt\right]$ captures utility from consumption, which the household chooses continuously. The term $\mathbb{E}_0\left[e^{-\rho\tau}(v^*(x_\tau) - \bar{\varepsilon})\right]$ captures discrete adjustment, which the household chooses at time τ (a stopping time). These discrete adjustments form an optimal stopping (option value) problem. Function v^* denotes the optimal value function conditional on adjusting, which also requires effort cost $\bar{\varepsilon}$. Adjustment takes the form of either mortgage prepayment or refinancing. Note that the mortgage interest rate remains constant when the household chooses a mortgage prepayment, while refinancing resets the mortgage interest rate to $r_t + \omega^m$.

Equation (7) highlights that the household faces a simultaneous optimal control problem plus an optimal stopping problem. The household continuously chooses consumption c_t, and also possesses the option to discretely rebalance its asset allocation across liquid wealth and illiquid home equity. To capture these dual decisions, the value function in equation (7) can

[15] Augenblick (2018) estimates that the division between "now" and "later" is approximately 2 hours. Using fMRI data, McClure et al. (2007) find a 50% discount rate over one hour for food rewards. More generally, Augenblick (2018) and Augenblick and Rabin (2019) show that essentially all discounting occurs within one week. See also DellaVigna (2018) for a discussion.

also be expressed as a Hamilton-Jacobi-Bellman Quasi-Variational Inequality (HJBQVI).[16] Starting with compact notation to highlight the general structure of variational inequalities:

$$\rho v(x) = \max\big\{ \max_c u(c) + (\mathcal{A}v)(x) \ , \ \rho(v^*(x) - \bar{\varepsilon})\big\}. \tag{8}$$

Operator $\mathcal{A}$ is an infinitesimal generator, which we will define momentarily by writing out equation (8) in a less compact fashion. The left branch of equation (8) captures the optimal control problem while the right branch captures the optimal stopping problem. If it is not optimal to adjust, the left branch imposes that value function $v(x)$ satisfies a standard HJB equation $\rho v(x) = \max_c u(c) + (\mathcal{A}v)(x)$, and the right branch imposes that $v(x)$ is larger than the value of adjusting: $v(x) \geq v^*(x) - \bar{\varepsilon}$. If it is optimal to adjust, the right branch imposes that the value function equals the value of adjusting: $v(x) = v^*(x) - \bar{\varepsilon}$.

Expanding the operator $\mathcal{A}$ to explicitly show the Poisson risks faced by the household, the HJBQVI can be written out fully as follows:

$$\begin{aligned}\rho v(x) = \max\Bigg\{ \max_c \Bigg\{ & u(c) + \frac{\partial v(x)}{\partial b}(y + rb + \omega^{cc} b^- - (r^m + \xi)m - c)\Bigg\} \\ & - \frac{\partial v(x)}{\partial m}(\xi m) \\ & + \sum_{y' \neq y} \lambda^{y \to y'}\big[v(b, m, y', r^m, r) - v(b, m, y, r^m, r)\big] \\ & + \sum_{r' \neq r} \lambda^{r \to r'}\big[v(b, m, y, r^m, r') - v(b, m, y, r^m, r)\big] \\ & + \lambda^R\big[v^R(x) - v(x)\big] \\ & + \lambda^F\big[v^*(x) - (v(x) - \bar{\varepsilon})\big], \\ & \rho(v^*(x) - \bar{\varepsilon})\Bigg\}.\end{aligned} \tag{8'}$$

Each of the seven rows of equation (8′) reflects the value function's dependence on, respectively, liquid wealth b (row one), mortgage level m (row two), income state y (row three), interest rate state r (row four), retirement (row five), forced adjustment (row six), and discrete adjustment (row seven). In rows three and four of the equation, which correspond

[16]See Bensoussan and Lions (1982, 1984) and Bardi and Capuzzo-Dolcetta (1997). For additional details and a discussion of more economic applications, see http://benjaminmoll.com/liquid_illiquid_numerical/, and McKay and Wieland (2019) and Guerrieri et al. (2020). Relative to our formulation (8), the mathematics literature studying HJBQVIs typically uses somewhat different but equivalent notation, for example

$$0 = \min\big\{\rho v(x) - \Big[\max_c u(c) + (\mathcal{A}v)(x)\Big] \ , \ v(x) - (v^*(x) - \bar{\varepsilon})\big\}.$$

We use the formulation with the max operator (8) because it is more economically intuitive.

respectively to the income process and the interest rate process, we use notation $\lambda^{s\to s'}$ to denote the transition rate from state s to state s'. λ^R is the transition rate into retirement, and λ^F is the rate at which households are forced to adjust their mortgage.

Value Functions ($\beta < 1$)**.** We now introduce naive present bias. The naive household incorrectly perceives that all future selves will discount exponentially ($\beta = 1$). Thus, the value function $v(x)$ that solves equation (7) – or equivalently (8) – characterizes the naive IG household's perceived value function starting in the next instant. For this reason, we will refer to $v(x)$ as the *continuation-value function.* The *current-value function* characterizes the household's perceived value of future utility flows in the current period. Since the current self discounts all future selves by β, the current-value function of the naive IG household is given by:

$$\begin{aligned} w(x) &= \max\left\{\beta v(x),\ w^*(x) - \bar{\varepsilon}\right\} \quad \text{with} \\ w^*(x) &= \max\left\{w^{prepay}(x), w^{refi}(x)\right\} \\ w^{prepay}(x) &= \max_{b',m'}\ w(b', m', y, r^m, r) \quad \text{s.t. prepayment constraint (4) holds} \\ w^{refi}(x) &= \max_{b',m'}\ w(b', m', y, r + \omega^m, r) \quad \text{s.t. refinancing constraint (5) holds} \end{aligned} \tag{9}$$

In the first line of equation (9), the left branch captures the current-value function if the household does not adjust its mortgage, and the right branch captures the current-value function if the household chooses to adjust its mortgage. Importantly, the utility flow of the current self does not show up in equation (9). Because each self lives for a single instant of dt, no individual self's utility flow has a measurable effect on the overall value function. Further discussion is provided in Appendix B, which derives the current-value function of the naive IG household as the continuous-time limit of a discrete-time model.

It is worth emphasizing again that, with naive present bias, the perceived value function v on the right-hand side of (9) *is the value function of a* $\beta = 1$ *household*, i.e. the one that solves (8). This property of naive present bias is the key reason for its tractability. In particular, it implies that both theoretical and computational approaches can use the following two-step procedure: first, solve the value function v of an exponential $\beta = 1$ household from (8); second, find the value function of a present-biased $\beta < 1$ household immediately from (9).

Policy Functions. Households make two choices in the model: they choose consumption continuously, and they have the option to adjust their mortgage discretely. Introducing notation for these policy functions, let $c : x \to [0, \infty)$ denote a household's consumption policy in state $x = (b, m, y, r^m, r)$. Mortgage adjustment consists of two nested decisions. First, conditional on adjustment the household chooses its new mortgage level m' and liquid wealth b'. Let $m' : x \to [0, \theta h]$ and $b' : x \to [\underline{b}, \infty)$ denote the household's optimal mortgage and liquid wealth choice, conditional on adjustment. Second, the household chooses whether or not to adjust. Let $\mathfrak{R} : x \to \{0, 1, 2\}$ denote whether a household finds it optimal to not adjust ($\mathfrak{R} = 0$), prepay ($\mathfrak{R} = 1$), or refinance ($\mathfrak{R} = 2$).

2.3 Procrastination

There is a large literature documenting that households are slow to refinance their mortgage after interest rate declines (e.g., Keys et al., 2016; Johnson et al., 2019; Andersen et al., 2020).[17] Refinancing involves a series of up-front effort costs, such as negotiating with mortgage brokers and filling out paperwork, in exchange for long-run financial benefits. Households with naive present bias will delay completing these sorts of immediate-benefit delayed-reward tasks, instead deferring them for future selves (O'Donoghue and Rabin, 1999, 2001; DellaVigna and Malmendier, 2004). In this way, naive present bias provides one natural motivation for refinancing inertia: *procrastination*.

Keys et al. (2016) provide direct evidence of procrastination as an important channel through which refinancing inertia arises. The financial calculations involved in refinancing are complex (Agarwal et al., 2013), and refinancing generates a range of non-pecuniary short-term costs in exchange for uncertain long-term benefits. This creates an environment where a variety of psychological factors – such as trust in the financial system, financial illiteracy, sticky information, attention costs, and bounded rationality – underlie the effort costs that drive procrastination.[18] Our goal here is to provide a simple and transparent model that captures the intuition for how such cognitive costs can interact with present bias to produce refinancing inertia.

The key model ingredient that generates procrastination for $\beta < 1$ households is the effort cost that households face to discretely adjust their mortgage. While the setup with a constant effort cost that we spelled out in Sections 2.1 and 2.2 already gives rise to procrastination, we here generalize this setup slightly by assuming that the effort cost is stochastic. This stochasticity captures the idea that households face occasional windows of time in which the marginal cost of making effortful budgeting adjustments is lower than normal. As we explain below, a stochastic effort cost will generate a particular form of procrastination, namely "Calvo-style" procrastination, that is not only tractable but also an accurate representation of household refinancing behavior (Andersen et al., 2020).

[17] There is also evidence that some households refinance too quickly (Agarwal et al., 2013, 2016). Our model cannot capture this behavior.

[18] For references to some of these behavioral considerations, see respectively Johnson et al. (2019); Agarwal et al. (2017); Mankiw and Reis (2002); Sims (2003); Woodford (2003); Gabaix (2019).

Assumption 1 *The effort cost follows a two-state Poisson process,* $\varepsilon_t \in \{\underline{\varepsilon}, \bar{\varepsilon}\}$ *with*

$$\bar{\varepsilon} > \frac{1}{\beta}\underline{\varepsilon} > 0.$$

The effort cost ε_t *switches from* $\bar{\varepsilon}$ *to* $\underline{\varepsilon}$ *at rate* ϕ*, and from* $\underline{\varepsilon}$ *to* $\bar{\varepsilon}$ *at rate* $\underline{\phi} \to \infty$*.*

Assumption 1 implies that ε_t sits at $\bar{\varepsilon}$, and occasionally drops to $\varepsilon_t = \underline{\varepsilon}$ for a single instant. Though stylized with a two-state process for tractability, this captures the sorts of stochastic life events, such as a free weekend or a cancelled afternoon meeting, in which the household becomes more willing to complete chores because the opportunity cost of time has temporarily fallen.

What is critical about these sorts of stochastic windows of availability is that they typically have explicit end dates. For example, a free weekend may represent a low-cost period for a household, but that window closes on Sunday night. These sorts of deadlines are forcing mechanisms that encourage present-biased households to complete effortful tasks, because even present-biased households will want to take advantage of relatively low-cost periods before they come to an end (see e.g. Carroll et al., 2009; Allcott et al., 2021).

Assumption 1 makes the simplification that these low-cost windows last for exactly one instant dt. This simplification maintains the stationarity of our continuous-time model (avoiding the need to include time as a state variable), but it is not critical to the results. The essential feature of these low-cost windows is that they have a defined expiration date.

This simple generalization of our model to a stochastic effort cost requires us to append our model equations in a few places. Appendix B.3 spells out the full set of equations. For example, in addition to the current-value of a $\beta < 1$ household in the high-effort-cost state defined in equation (9), there is now an analogous equation for a household in the low-effort-cost state:

$$\underline{w}(x) = \max\left\{\beta v(x),\ w^*(x) - \underline{\varepsilon}\right\}. \tag{10}$$

Intuitively, since the low-cost state only lasts for an instant (Assumption 1), the household either takes advantage of adjusting its mortgage at the lower effort cost $\underline{\varepsilon}$ in which case its value is $w^*(x) - \underline{\varepsilon}$, or else the household reverts to the high-cost state and its value is $\beta v(x)$. Equation (10) highlights the cost of not refinancing when $\varepsilon = \underline{\varepsilon}$ — the low-cost period is lost and the effort cost reverts back to $\bar{\varepsilon}$. For future reference, we will denote by $\underline{\mathfrak{R}}(x), \underline{m}'(x)$, and $\underline{b}'(x)$ the corresponding refinancing policy function in the low-effort-cost state (i.e., the prepayment or refinancing decisions corresponding to (10)), while $\mathfrak{R}(x), m'(x)$, and $b'(x)$ continue to denote the analogous policy function when the effort cost is high.

Next, we make an assumption so that the effort cost only matters for $\beta < 1$ households:

Assumption 2 $\bar{\varepsilon}$ *and* $\underline{\varepsilon}$ *are vanishingly small.*

Assumption 2 represents the idea that households typically consider refinancing to be a nuisance, but not costly in an economically meaningful sense. By making the effort cost arbitrarily small, the effort cost is inconsequential for the behavior of $\beta = 1$ households.

However, this trivial effort cost becomes important when *interacted* with present bias. When $\beta < 1$, the small effort cost is sufficient to generate procrastination.[19] This is because naive present-biased households will always choose to delay the task of refinancing (for one instant in expectation) whenever $\varepsilon_t = \bar{\varepsilon}$. The perceived benefit of procrastinating is that the effort cost of adjustment gets pushed into the future, where it is discounted by β. When $\varepsilon_t = \bar{\varepsilon}$, the perceived cost of delaying for one instant is infinitesimal. So, naive present-biased households will continually procrastinate when $\varepsilon_t = \bar{\varepsilon}$.

For $\beta < 1$ households, procrastination persists until the household stochastically enters a low-cost window and the effort cost ε_t momentarily drops to $\underline{\varepsilon}$. Now, there is an explicit cost to waiting — any further procrastination causes the low-cost window to expire and the effort cost to revert to $\bar{\varepsilon} > \frac{1}{\beta}\underline{\varepsilon}$. Since the effort cost of completion now, $\underline{\varepsilon}$, is less than the discounted effort cost of completion next instant, $\beta\bar{\varepsilon}$, this fleeting opportunity incentivizes the present-biased household to stop procrastinating. This is formalized in Proposition 2 below.[20]

3 The Effect of Present Bias on Policy Functions

We are now prepared to describe the effect of present bias on the consumption and mortgage adjustment decisions. In order to characterize the policy functions of a present-biased household relative to a standard exponential household, we use hat-notation to denote the policy functions of an otherwise-identical household that has $\beta = 1$. Accordingly, the policy functions denoted by hats are the policy functions that the naive household perceives that all future selves will adopt.

Consumption. Households earn utility from consumption, and present-biased households want to bring utility forward in time. This means that present bias has a direct effect on households' consumption-saving decisions. Specifically, present-biased households overconsume by factor $\beta^{-\frac{1}{\gamma}}$:

[19] Of course, when $\beta = 1$ it is possible to rationalize refinancing inertia by making other assumptions about effort cost ε_t, for example by making $\bar{\varepsilon}$ arbitrarily large.

[20] As this discussion makes clear (and we confirm in Proposition 2 below), our baseline model in Sections 2.1 and 2.2 with a constant effort cost $\bar{\varepsilon}$ would generate indefinite refinancing procrastination by $\beta < 1$ households. Perhaps surprisingly, this is true even though this effort cost is arbitrarily small, $\bar{\varepsilon} \to 0$ (Assumption 2). Therefore, an additional rationale for extending our model to feature a stochastic effort cost, besides increased realism (low-cost windows like weekends and cancelled meetings do exist), is that it generates $\beta < 1$ households who refinance.

Proposition 1 (Continuous Control)

1. *For all $b > \underline{b}$, the household sets $c(x) = \beta^{-\frac{1}{\gamma}}\widehat{c}(x)$.*
2. *For $b = \underline{b}$, the household sets $c(x) = \min\left\{\beta^{-\frac{1}{\gamma}}\widehat{c}(x),\ y + (r + \omega^{cc})\underline{b} - (r^m + \xi)m\right\}$.*

The proof of Proposition 1 makes use of an important intermediate step, which we state first and which is proved in Appendix B:

Lemma 1 *When the borrowing constraint does not bind, $b > \underline{b}$, consumption is defined implicitly by the first-order condition:*

$$u'(c(x)) = \beta \frac{\partial v(x)}{\partial b}, \tag{11}$$

where the continuation-value function v is equal to the value function of an exponential $\beta = 1$ household and solves (7) or equivalently (8).

Equation (11) is a first-order condition: consume until the marginal utility of consumption equals the marginal value of liquid wealth. For $\beta = 1$, the standard continuous-time first-order condition of $u'(c(x)) = \frac{\partial v(x)}{\partial b}$ is recovered. Generically, the additional discount factor β arises in equation (11) because the present-biased household discounts the value of future consumption (and hence current wealth) by β.[21] It is worth emphasizing again that naiveté implies that the continuation-value function v in (11) is the value function of an exponential $\beta = 1$ household. This means that, similarly to the value function, one can recover the consumption policy function with a two-step procedure: first, solve the value function v of an exponential $\beta = 1$ household from (8); second, find the consumption policy function of a present-biased $\beta < 1$ household from (11) (with an additional condition when $b = \underline{b}$).

Proof of Proposition 1. Let $\widehat{c}(x)$ denote the consumption function that the naive IG household expects all future selves to adopt ($\beta = 1$). Expanding equation (11) under CRRA utility, we have:

$$c(x)^{-\gamma} = \beta \frac{\partial v(x)}{\partial b}, \text{ and } \widehat{c}(x)^{-\gamma} = \frac{\partial v(x)}{\partial b}.$$

Rearranging gives $c(x) = \beta^{-\frac{1}{\gamma}}\widehat{c}(x)$. This holds as long as $b > \underline{b}$. For $b = \underline{b}$, overconsumption will be restricted if the borrowing constraint binds (see equation (3)). ■

[21] While (11) looks like a standard discrete-time first-order condition, it is important to note that the interpretation is different. In particular, β is *not* the standard exponential discrete-time discount factor; instead, β is the discount factor between now and one instant from now, see (6), with $\beta = 1$ corresponding to exponential discounting, i.e. no present bias. Another way of putting this is: in discrete time the quasi-hyperbolic discount function is $1, \beta\delta, \beta\delta^2, \beta\delta^3, ...$ so that δ is the standard exponential discount factor, but (11) features β and not δ.

Proposition 1 provides a tractable formula that relates the IG household's consumption to that of a standard exponential household. This can be used to construct an Euler equation for the IG household:

Corollary 1 (Maxted (2020)) *Let $r_t(b_t)$ denote the household's effective borrowing cost: $r_t(b_t) = r_t$ if $b_t \geq 0$, and $r_t(b_t) = r_t + \omega^{cc}$ if $b_t < 0$. Whenever $c(x_t)$ is locally differentiable in b, consumption obeys the following Euler equation:*

$$\mathbb{E}_t \frac{du'(c(x_t))/dt}{u'(c(x_t))} = \left[\rho + \gamma \left(1 - \beta^{\frac{1}{\gamma}}\right) \frac{\partial c(x_t)}{\partial b}\right] - r_t(b_t). \tag{12}$$

Proof. Appendix C extends the proof of Maxted (2020) to our environment. ■

The growth rate of marginal utility is given by $\mathbb{E}_t \frac{du'(c(x_t))/dt}{u'(c(x_t))}$, and the term in brackets in equation (12) can be interpreted as the household's effective discount rate at time t. When $\beta = 1$, equation (12) reduces to a standard Euler equation: $\mathbb{E}_t \frac{du'(c(x_t))/dt}{u'(c(x_t))} = \rho - r_t(b_t)$. For $\beta < 1$, the household's effective discount rate is increasing in the household's instantaneous MPC, $\frac{\partial c(x_t)}{\partial b}$. Intuitively, overconsumption will have a larger effect on the growth rate of marginal utility when consumption itself is sensitive to b_t.

For $\beta < 1$, an important consequence of equation (12) is that the household's effective discount rate varies over the state space. In particular, households with a higher instantaneous MPC will consume more impatiently. Since the consumption function is concave in liquid wealth, households near $\underline{b}$ will act more impatiently, while households with plentiful liquidity will act more patiently.

Mortgage Adjustment. Next we characterize the effect of present bias on the household's mortgage adjustment decision. To this end, recall that $\mathfrak{R}(x) \in \{0, 1, 2\}$ and $\underline{\mathfrak{R}}(x) \in \{0, 1, 2\}$ denote the household's decision to not adjust, prepay, or refinance in the high- and low-effort-cost states, and $m'(x), b'(x), \underline{m}'(x)$, and $\underline{b}'(x)$ denote its adjustment targets conditional on adjusting. Unlike consumption, present bias has a muted impact on these adjustment decisions. In particular, the only way that $\beta < 1$ affects the mortgage adjustment decision is through procrastination.[22] This is formalized in the following proposition:

Proposition 2 (Optimal Stopping)

1. *Adjustment targets m' and b' are independent of β. Thus, $m'(x) = \widehat{m}'(x)$, $b'(x) = \widehat{b}'(x)$, $\underline{m}'(x) = \widehat{\underline{m}}'(x)$, and $\underline{b}'(x) = \widehat{\underline{b}}'(x)$ for all x.*

[22]If we were to set effort cost $\varepsilon_t \equiv 0$ then present bias would have no effect on the household's mortgage adjustment decision.

2. (a) *For $\beta = 1$, the refinancing policy function $\mathfrak{R}(x)$ converges pointwise to $\underline{\mathfrak{R}}(x)$ as the effort cost vanishes. This effectively means that the $\beta = 1$ household's mortgage adjustment behavior does not depend on the state of the effort cost.*

   (b) *For $\beta < 1$ and $\varepsilon = \bar{\varepsilon}$, $\mathfrak{R}(x) = 0$ for all x. This means that the present-biased household procrastinates and will not adjust its mortgage when $\varepsilon = \bar{\varepsilon}$.*

   (c) *For $\beta < 1$ and $\varepsilon = \underline{\varepsilon}$, $\underline{\mathfrak{R}}(x)$ converges pointwise to $\widehat{\underline{\mathfrak{R}}}(x)$ as the effort cost vanishes. This effectively means that the present-biased household does not procrastinate when $\varepsilon = \underline{\varepsilon}$.*

Proof. See Appendix C. The intuition for clause 1 is that the current self composes only an infinitesimal part of the household's overall value function. This implies that $w^{prepay}(x)$ and $w^{refi}(x)$ in (9) can be rewritten as:

$$w^{prepay}(x) = \max_{b',m'} \; \beta v(b', m', y, r^m, r) \quad \text{s.t. prepayment constraint (4) holds}$$
$$w^{refi}(x) = \max_{b',m'} \; \beta v(b', m', y, r + \omega^m, r) \quad \text{s.t. prepayment constraint (5) holds}$$

Since maximizing βv is equivalent to maximizing v, the same (b', m') will be optimal for the $\beta < 1$ household and the $\beta = 1$ household. The intuition for clause 2 was discussed above in Section 2.3. ■

Clauses 2b and 2c of Proposition 2 state that present-biased households refinance only when they are in a refinancing region of the state space *and* they experience a low-cost window (weekends, cancelled meetings, etc). Recall from Assumption 1 that these low-cost windows occur at Poisson rate ϕ. Therefore the model with naive present bias reproduces the state- plus time-dependent refinancing behavior documented in Andersen et al. (2020).

A Comparison: Consumption versus Mortgage Adjustment. The juxtaposition of Propositions 1 and 2 highlights the subtleties of present bias' impact on household balance-sheet decisions. In our model, present bias directly affects the consumption decision, whereas present bias only affects the mortgage adjustment decision through procrastination.

Present bias has a differential impact on these decisions because consumption and procrastination are "small" decisions (flow decisions) while discrete adjustment is a "large" decision (a stock decision). The current self wants to overconsume in the moment, but this overconsumption has only an infinitesimal effect on the household's balance sheet. Similarly, procrastination is expected to last only for an instant. The same is not true of the mortgage adjustment decision. This decision discretely adjusts the household's asset allocation between liquid wealth and illiquid home equity. Since each self only lasts for an instant, any short-term benefits from myopia-driven refinancing are dominated by the accumulation of costs borne across all future selves. In short, naive present bias causes a persistent accumulation of small mistakes, not the intermittent occurrence of large mistakes.

The proofs of Propositions 1 and 2 rely on two assumptions: naiveté and continuous time. The assumption of naiveté is critical. Naiveté implies that the current-value function w is related to the value function v of an exponential household (see equation (9)).[23] The assumption of continuous time – where each self lasts for a single instant – is mathematically convenient but not quantitatively necessary. Propositions 1 and 2 will be robust so long as the temporal division between the present and the future is relatively short (e.g., one week), meaning that each self composes a negligible part of the overall value function.[24]

The Effect of Present Bias on Refinancing. There is an informal intuition in the literature that present bias increases the propensity for households to extract home equity in order to finance near-term consumption (see e.g. Mian and Sufi, 2011; Campbell, 2013). Our model illustrates that the effect of present bias on refinancing is more complex. When effort cost $\varepsilon = \underline{\varepsilon}$, Proposition 2 says that present bias has no effect on the refinancing decision at any given point x. When $\varepsilon = \bar{\varepsilon}$, procrastination slows down the rate at which present-biased households refinance. Both of these effects counter the intuition that present bias increases cash-out refinancing. However, present bias causes a build-up of credit card debt due to overconsumption. This incentivizes home-equity extractions as a means of converting costly credit card debt to cheaper mortgage debt. In summary, present bias inhibits refinancing at any given point x. But, present bias changes the distribution of households over the state space in a way that encourages cash-out refinancing.

4 Calibration and Steady State Household Behavior

4.1 Externally Calibrated Parameters

We begin by describing the model's externally calibrated parameters. These parameter choices are summarized in Appendix Table 5.

[23]See Maxted (2020) for a discussion of sophistication versus naiveté under IG preferences.

[24]For intuition, consider a discrete-time model where each self lives for one week. Let $\delta = \exp\left(\frac{-\rho}{52}\right)$. Assume that each self consumes a constant amount, $\bar{c}$, in each period. Each self has a current value of $u(\bar{c}) + \frac{\beta\delta}{1-\delta}u(\bar{c})$, meaning that the utility of each self composes a share $1/\left(1 + \frac{\beta\delta}{1-\delta}\right)$ of the total value function. Under our benchmark calibration with $\beta = 0.83$ and $\rho = 0.97\%$ (Section 4.2) so that $\delta = \exp\left(\frac{-\rho}{52}\right) = 0.9998$, this share equals 0.02% of the total value function.

Home Owners in the 2016 SCF. For many of our calibration targets we use the 2016 Survey of Consumer Finances (SCF) wave. We often express these targets relative to a measure of households' permanent income for which – following Kennickell (1995), Kennickell and Lusardi (2004), and Fulford (2015) – we use the SCF's question about "normal income." To align the SCF sample with the households in our model, we impose the following sample restrictions: households must have a head who is in the labor force, aged 25-66, owns a home, and possesses a credit card. Since housing h is fixed in our model, we further condition on households with a home value to permanent income ratio between the 25th and 75th percentile to limit empirical heterogeneity in home values. The average after-tax permanent income for households in our SCF sample is roughly \$100,000. See Appendix A.2 for details.

Income. We assume that the Poisson income process takes one of three values $y_t \in \{y_L, y_M, y_H\}$. We limit the model to three income states – low, middle, and high – to avoid complicating the exposition of the model.[25]

Our calibration of the income process follows Guerrieri and Lorenzoni (2017), who assume that log-income is an AR(1) process at a quarterly frequency.[26] We first convert this quarterly AR(1) process to a continuous-time Ornstein-Uhlenbeck (OU) process using the method described in Appendix A.3, and then discretize the OU process with a three-state Poisson process using a finite difference method. We normalize mean income to 1 and set the three income states equal to $y_t \in \{0.75, 0.98, 1.28\}$, corresponding to $-1, 0, +1$ standard deviations of the estimated log-income process.[27]

Interest Rates. We view r_t as a longer-term interest rate, and assume that the monetary authority adjusts short rates in the background to generate these movements in the long rate. Our model aims to capture larger movements in the federal funds rate, such as those in 2001, 2008, and 2019-20, which have economically significant effects on long rates and hence on mortgage refinancing. The focus of our paper is not on smaller, fine-tuning, movements in the federal funds rate that do not significantly affect mortgage rates. Rather than treating all shocks to r_t as unexpected "MIT shocks," households in our model have rational expectations about the data-generating process for r_t.

In order to calibrate household interest rate expectations we estimate an AR(1) process

[25]Our model includes a wedge between the interest rate on borrowing versus saving (i.e., a "soft constraint"). In consumption-saving models such as this, three income states are the minimum number of states that are needed to generate a mass of households who borrow, a mass of households at the soft constraint, and a mass of households who save (Achdou et al., 2021).

[26]Using data from Floden and Lindé (2001), Guerrieri and Lorenzoni (2017) calibrate this process with persistence $\rho = 0.967$ and variance $\sigma^2 = 0.017$.

[27]Under this calibration, 31% of households are low income, 39% are middle income, and 31% are high income. The expected persistence of the low, middle, and high income states are 1.6 years, 1 year, and 1.6 years, respectively.

via maximum likelihood estimation using weekly data on the yield of 10-year TIPS from 2003 – 2019. We first convert this weekly AR(1) process to a continuous-time OU process (with rate of mean reversion 0.29 and volatility 0.63%), and then discretize it into a four-state Poisson process with states $r_t \in \{-1\%, 0\%, 1\%, 2\%\}$ using a finite difference method. We view this calibration as consistent with the current low-interest environment.

We set the credit card wedge ω^{cc} to 10.3% to capture the difference between the commercial bank interest rate charged on credit cards and the 10-year treasury yield from 2015 – 2017.[28] We set $\omega^m = 1.7\%$ to capture the difference between the average 30-year fixed rate mortgage and the 10-year treasury yield from 2015 – 2017.

Assets and Liabilities. Using the sample of home owners in the 2016 SCF, we estimate an average home value to permanent income ratio of 3.1 (roughly $310,000) and therefore set $h = 3.1$. We set θ, the maximum LTV ratio, equal to 0.8. Although this is a tight restriction on the maximum LTV allowed for first-time homebuyers, it is consistent with the maximum LTV available to households conducting a cash-out refinance.[29]

Mortgages are paid down at rate $\xi = 3.5\%$, which generates a 20-year half-life for mortgages. As was discussed above, our model of mortgage repayment is non-standard because mortgage payments are not constant. We choose a 20-year half-life for mortgage paydowns so that the mortgage payment required by large mortgages is not exceedingly onerous. We set the fixed cost of refinancing to 3% of the average outstanding mortgage principal, resulting in $\kappa^{refi} = 0.05$ (approximately $5,000).[30] For numerical tractability, we also impose a small cost to prepaying mortgages of 0.1% of the average outstanding mortgage principal ($\kappa^{prepay} = 0.002$, or approximately $200).

We set the credit card borrowing limit $\underline{b}$ to one third of permanent income. This is consistent with reported credit card borrowing limits in the SCF, and in line with typical choices in the literature.[31]

Preferences. We set the coefficient of relative risk aversion γ equal to 2, which is a standard calibration in the consumption-saving literature. Discount function parameters ρ and β are calibrated internally to match the home equity and credit card debt accumulation observed in the 2016 SCF — see Section 4.2 below.

We choose a Poisson rate governing procrastination of $\phi = -\ln(0.5)$. This implies that

[28] Credit card interest rates are collected from the Federal Reserve's G.19 release.

[29] See Greenwald (2018) for data on realized LTVs for first-time homebuyers and for cash-out refinances.

[30] The Federal Reserve's website on refinancing suggests that refinancing costs roughly 3% of outstanding principle. The average LTV in our model is 0.54, suggesting $\kappa^{refi} = 0.03 \times (0.54h)$. This yields roughly $\kappa^{refi} = 0.05$. For details, see https://www.federalreserve.gov/pubs/refinancings/.

[31] For example, Kaplan et al. (2018) calibrate a borrowing limit of one times quarterly labor income.

there is a 50% probability that a household for whom it is optimal to refinance will do so within one year, consistent with Andersen et al. (2020).[32]

Other Structural Parameters. We set the rate of forced adjustment to $\lambda^F = \frac{1}{15}$, which approximates the moving rate of homeowners reported in the Current Population Survey's Annual Social and Economic Supplement for 2016. The retirement rate $\lambda^R = \frac{1}{30}$ so that households exist in our model for 30 years on average. We set the retirement income flow y^R equal to the minimum income level y_L, since $y_L = 0.75$ and a retirement replacement rate of 70-80% is a common benchmark. We assume that retired households are replaced by new households with mortgage $m_t = \theta h$ and liquid wealth b_t drawn from a uniform distribution with support $[0, y_L]$.

4.2 Internally Calibrated Parameters and Steady State

"Steady State." We start all policy counterfactuals from a "steady state" in which the interest rate has been permanently fixed at $r^* = 1\%$. This implies that all households in our steady state will have a mortgage interest rate of $r_t^m = r^* + \omega^m = 2.7\%$. The assumption that the interest rate has been fixed at 1% is formally just one possible sample path of the interest rate process. This assumption is helpful for pedagogy because it reduces the dimensionality of our steady state by eliminating heterogeneity in r_t^m.[33]

Calibration Cases. We present two benchmark calibration cases for the "steady state": an *Exponential Benchmark* with $\beta = 1$ and a *Present-Bias Benchmark* with $\beta < 1$. In equilibrium, the Exponential Benchmark features no procrastination while the Present-Bias Benchmark features procrastination (see Proposition 2). For conceptual clarity, we also present the *Intermediate Case* of $\beta < 1$ with no refinancing procrastination.[34] The Intermediate Case, with present bias but no procrastination, should be viewed as a transitional step between the Exponential Benchmark and the Present-Bias Benchmark.

We recalibrate the discount function for each of the three cases in order to reproduce the home equity and credit card debt accumulation observed in our 2016 SCF sample. In the SCF, we estimate an average LTV ratio of 0.54 and an average credit card debt to permanent income ratio of 0.09. Note that credit card debt appears to be underreported in the SCF

[32] Andersen et al. (2020) estimate that 84% of households are "asleep" each quarter, and $0.84^4 \approx 0.5$.

[33] We choose $r^* = 1\%$ as the steady state interest rate to ensure that we have ample policy space for studying interest rate cuts. We will study rate cuts from 1% to 0%, meaning that interest rates can still fall further (in particular, to -1%). Without room to fall further, there would be no option value to delaying refinancing. Households would rush to refinance in order to lock in the lowest possible interest rate.

[34] This intermediate calibration corresponds to the rate at which the effort cost ε_t flicks to $\underline{\varepsilon}$ becoming unboundedly large, $\phi \to \infty$.

(Zinman, 2009). We therefore adjust reported credit card borrowing upward by a factor of 1.5, following the methodology of Beshears et al. (2018, Appendix C).

Table 1 presents the discount function calibration. In all three cases, ρ is calibrated to match the average LTV of 0.54. For the $\beta < 1$ cases, we calibrate β to match the average credit card debt to income ratio of 0.09.[35] For readers who are uncomfortable with our 1.5-times adjustment to SCF credit card borrowing, we also estimate that 60% of households in our SCF sample report carrying a credit card balance from one month to the next. Table 1 shows that this feature is qualitatively matched by our $\beta < 1$ calibrations, but not by our Exponential Benchmark. Indeed, we would calibrate an even lower β value if we were to set β to reproduce the share of households with revolving credit card debt.

	Data	Exponential Benchmark	Intermediate Case	Present-Bias Benchmark
Discount Function				
β	-	1	0.70	0.83
ρ	-	1.47%	0.64%	0.97%
Calibration Targets				
LTV	0.54	**0.54**	**0.54**	**0.54**
Avg. C.C. Debt	0.09	0.03	**0.09**	**0.09**
Share C.C. Debt > 0	60%	26%	52%	47%

Table 1: Internally Calibrated Parameters.

Notes: This table summarizes the calibration of the discount function for the three cases we study.

In the Exponential Benchmark, $\rho = 1.47\%$ matches the average LTV moment of 0.54. However, the Exponential Benchmark fails to generate the level of credit card borrowing observed in the data.[36] In the two $\beta < 1$ cases there are two parameters (ρ and β) to match the two empirical moments. In the Intermediate Case, we calibrate $\beta = 0.70$ and $\rho = 0.64\%$. In the Present-Bias Benchmark, we calibrate $\beta = 0.83$ and $\rho = 0.97\%$.

In all cases the long-run discount rate ρ is relatively low. This is largely driven by the steady state interest rate of $r^* = 1\%$. Additionally, the two $\beta < 1$ cases feature a lower ρ than the $\beta = 1$ case. The lower ρ calibration offsets $\beta < 1$ to ensure that households still accumulate enough home equity to match the LTV moment. The Present-Bias Benchmark features a higher level of β than the Intermediate Case because procrastination generates additional credit card borrowing: households are slow to use cash-out refinances to pay off accumulated credit card debt. A higher β is required to offset this effect and to match the credit card borrowing moment.

Compared to the heterogeneous-agent macro literature with exponential time preferences,

[35] Meier and Sprenger (2010) combine surveyed time preferences with administrative credit card data to provide direct evidence of the relationship between present bias and credit card debt.

[36] This conclusion echoes the results of Laibson et al. (2020).

our calibration features a relatively large credit card wedge of $\omega^{cc} = 10.7\%$. Despite this large and empirically realistic credit card wedge, our model with present bias nevertheless generates sizable borrowing. Relatedly, our calibration features a relatively low mortgage interest wedge of $\omega^m = 1.7\%$ and hence a relatively low wedge between this effective illiquid return (r_t^m) and the return on liquid saving (r_t). That is, our model with present bias does not require a large illiquid return to help generate sizable credit card borrowing. In contrast to models with exponential discounting – which typically require calibrating either a relatively low credit card wedge or a relatively high illiquid return to deliver empirically realistic levels of unsecured borrowing – we instead take the credit card wedge and the illiquid return spread "from the data" and internally calibrate the additional discount function parameter β to match the credit card debt levels observed empirically.

4.3 Steady State Household Behavior

Each household solves an optimal control problem augmented with an optimal stopping problem, and the steady state features cross-sectional heterogeneity in four variables: b_t, m_t, y_t, and ε_t. When characterizing the model's steady state we focus on the features that will be important for the macroeconomic policy results to follow. As we detail throughout this section, many of the equilibrium behaviors that differentiate the Present-Bias Benchmark from the Exponential Benchmark are consistent with an emerging set of empirical findings in the household finance literature that have, collectively, proven challenging for models with exponential discounting to replicate.

Phase Diagrams of the Household Balance Sheet. Figure 1 uses phase diagrams to describe the evolution of households over the state space. From left to right, each panel represents a different income state. From top to bottom, we show the Exponential Benchmark, the Intermediate Case, and the Present-Bias Benchmark. The horizontal axis of each panel is liquid wealth b and the vertical axis is the household's mortgage balance, expressed as the LTV ratio m/h. The red and blue shaded areas indicate discrete adjustment regions: red indicates cash-out refinancing, and blue indicates discrete mortgage prepayment.[37] In areas of non-adjustment, the arrows indicate how the household balance sheet evolves over time.

Looking first at the red regions, households choose to take a cash-out refinance when they have relatively low income (either y_L or y_M) and they are near the credit card borrowing limit of $\underline{b}$. During periods of lower income, the first margin on which households adjust is to decumulate liquid wealth. The second margin is credit card debt. Even with wedge ω^{cc}, the fixed cost of refinancing implies that taking on credit card debt can be optimal in the short-run if households perceive that there is a high probability of quickly repaying this

[37] In the steady state with constant interest rates, the only reason to refinance is to withdraw home equity.

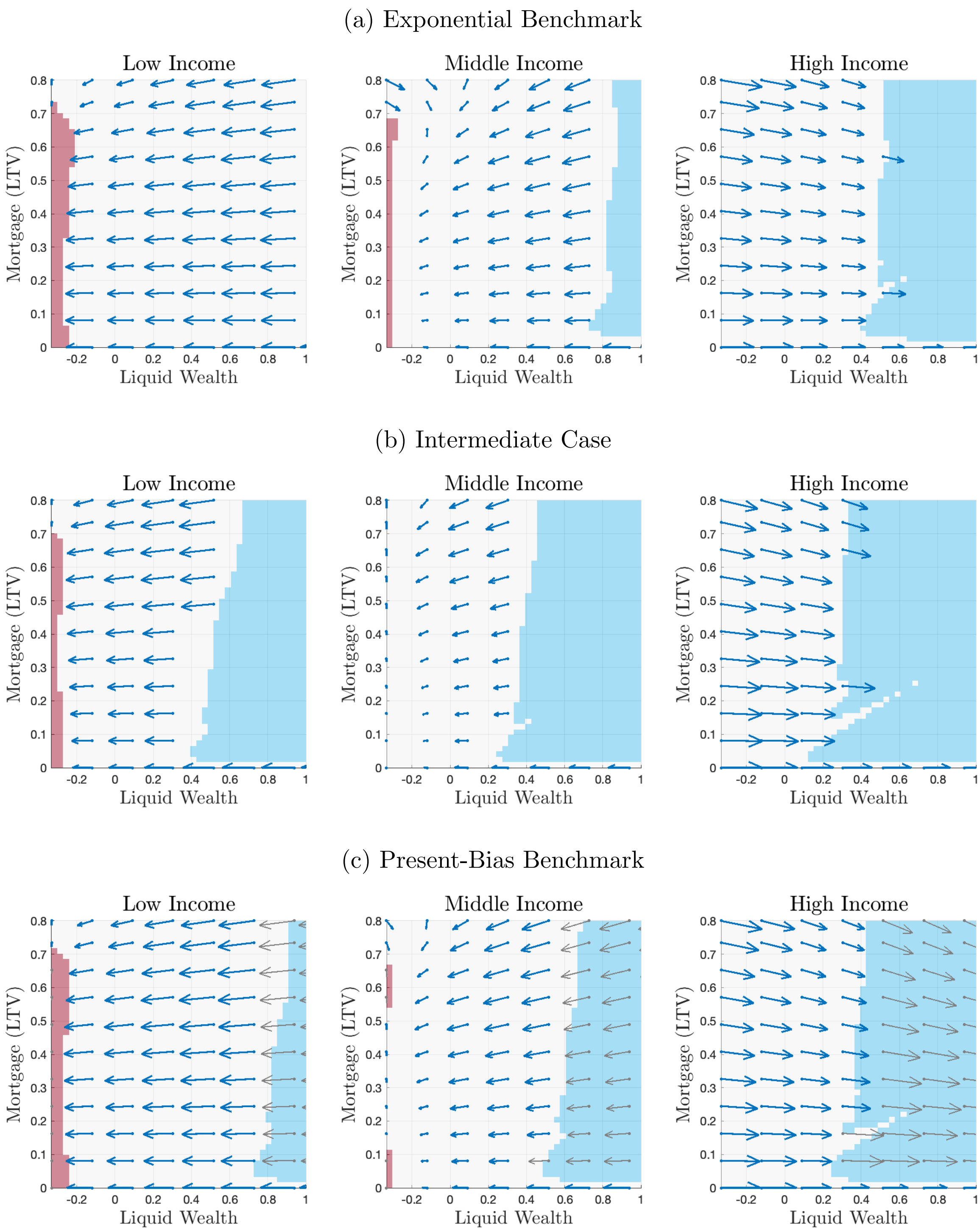


Figure 1: Steady State Phase Diagrams.

Notes: Arrows display the evolution of household balance sheets in non-adjustment regions. Red regions mark where the household chooses to conduct a cash-out refinance, and blue regions mark mortgage prepayment. See text for details.

debt. Cash-out refinances are the final margin that households draw upon, but only after accumulating sizable credit card debt.[38]

Looking next at the blue regions, households choose to prepay their mortgage once they have built up a buffer stock of liquid wealth. Having some liquid wealth is useful because it allows households to avoid taking on costly credit card debt during low-income spells. However, it is suboptimal to hold too much liquidity because the mortgage wedge ω^m implies that mortgage debt is more costly than the household's return on liquid wealth. Thus, high-liquidity households will use some of their accumulated liquidity to pay down their mortgage.

As shown in Proposition 2, all differences in the adjustment regions across the three calibrations are driven by variation in ρ. The top row (Exponential Benchmark) features the highest calibration of $\rho = 1.47\%$, the bottom row (Present-Bias Benchmark) calibrates $\rho = 0.97\%$, and the middle row (Intermediate Case) calibrates $\rho = 0.64\%$. Households who perceive themselves to be less patient (higher ρ) will be more willing to take a cash-out refinance and less willing to prepay their mortgage. The variation in the red cash-out region and blue prepayment region across the three calibrations follows accordingly.

The blue arrows indicate how household balance sheets evolve over the non-adjustment regions. The arrows always point downward due to mortgage principal repayment, as specified by equation (2). The arrows point either right or left to indicate liquid saving or dissaving, and the length of the arrow corresponds to the speed of evolution. Arrows point strongly leftward when $y_t = y_L$, indicating liquid dissaving. Arrows point strongly rightward when $y_t = y_H$, indicating liquid saving. When $y_t = y_M$ the arrows typically point to the left but are small, indicating slight dissaving by households.

The bottom row features gray arrows in the adjustment regions, while the top two rows do not. In the top two rows there is no procrastination. Households will "jump" as soon as they move into an adjustment region, and therefore households will never find themselves in the shaded regions. The bottom row features procrastination. This means that households will find themselves in the adjustment regions, and the gray arrows indicate how household balance sheets evolve when households procrastinate.

Steady State Consumption Dynamics. Figure 2 plots the steady state consumption function. Liquid wealth is on the horizontal axis. Each panel plots the consumption function for an LTV $\in \{0, 0.4, 0.8\}$.

We begin by explaining the Exponential Benchmark. Note that the curves do not always span the entire liquid-wealth axis. For example, the blue and orange curves do not extend to $\underline{b}$ in the Low Income panel, while the orange and green curves only extend to approximately

[38] Households with an LTV above approximately 0.7 do not conduct cash-out refinances, because these households have too little home equity to justify the fixed cost of refinancing.

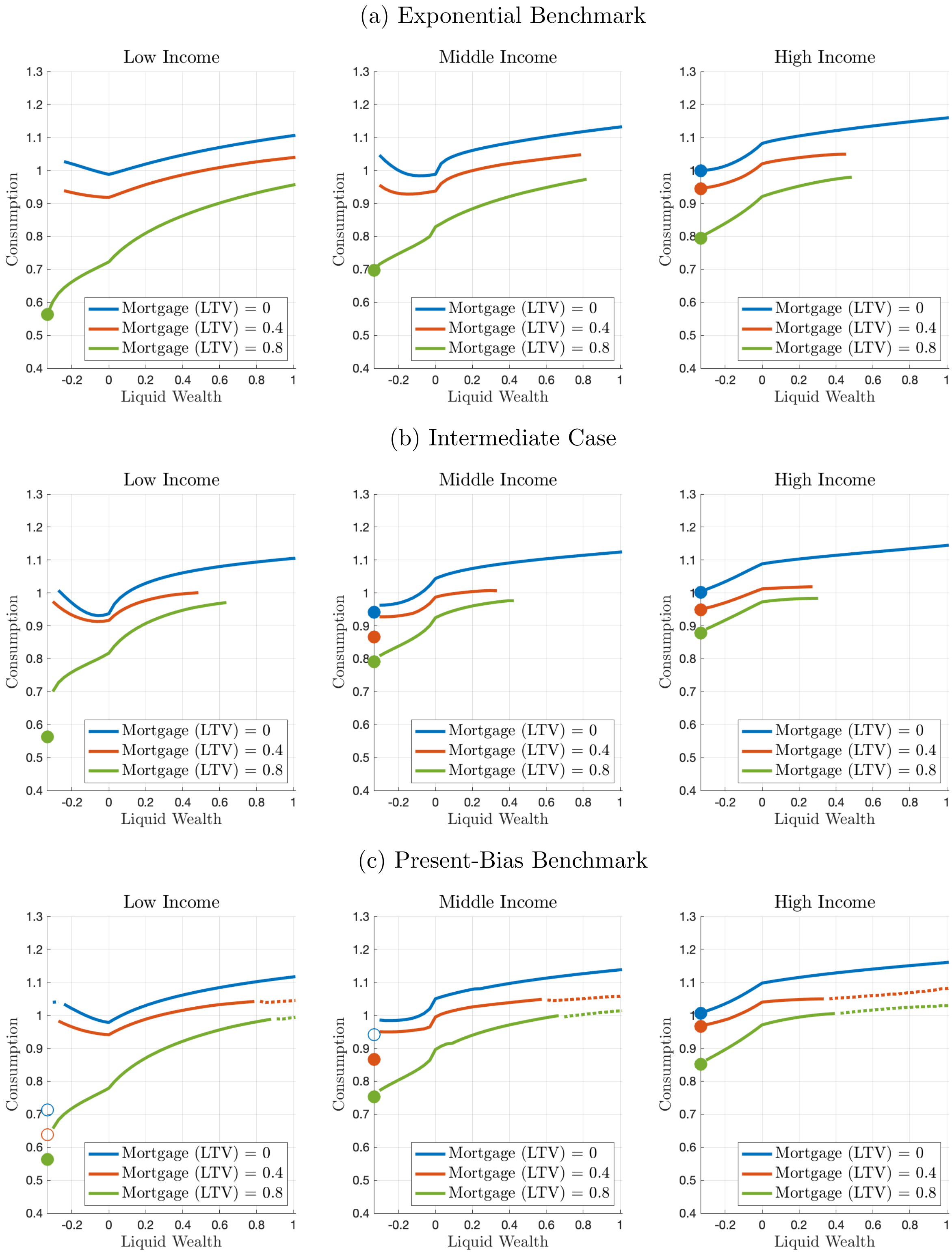


Figure 2: Steady State Consumption.

Notes: For the three calibration cases, this figure shows the steady state consumption function over income and liquid wealth for LTV $\in \{0, 0.4, 0.8\}$. See text for details.

$b = 0.5$ in the High Income panel. These missing areas reflect the adjustment regions. Households will never reach these parts of the state space.

The second row plots the consumption function for the Intermediate Case. The key effect of $\beta < 1$ is that the consumption function occasionally features a discontinuity at $\underline{b}$ (e.g., Low Income and LTV = 0.8). This discontinuity arises because the borrowing constraint restricts overconsumption (see constraint (3)). The consumption discontinuity is consistent with the evidence presented in Ganong and Noel (2019), who use high-frequency data to document that consumption drops sharply at the expiration of unemployment insurance.

The bottom row shows consumption in the Present-Bias Benchmark. Unlike the above two cases, dashed lines and open circles are used to mark the adjustment regions. Adjustment regions are now shown because procrastination implies that households can enter these regions. Procrastination strengthens the consumption discontinuity at $\underline{b}$. For example, households with $y_t = y_L$ and LTV = 0.4 experience a drop in consumption of approximately 30% if they fail to refinance before hitting $\underline{b}$. The reason for this large discontinuity is that households are naive about their procrastination. Once households are in an adjustment region they expect their next self to refinance. As a result, households choose consumption today in order to smooth consumption relative to where they expect to be following a cash-out refinance. Households do not anticipate hitting the borrowing constraint $\underline{b}$, creating a large downward discontinuity if the constraint does eventually bind.

Marginal Propensities to Consume (MPCs). Consumption behavior can also be investigated through MPCs. The marginal propensity to consume (MPC) over τ years is defined as follows (Achdou et al., 2021):

$$MPC_\tau(x) = \frac{\partial}{\partial b}\mathbb{E}\left[\int_0^\tau c(x_t)dt \mid x_0 = x\right]. \tag{13}$$

The top panel of Figure 3 plots the quarterly MPC out of \$1,000 as a function of liquid wealth b, averaging over the income and mortgage dimensions.[39] All three calibrations are shown, but the Intermediate Case is presented with a dashed line since it is a transitional step between our two benchmarks.

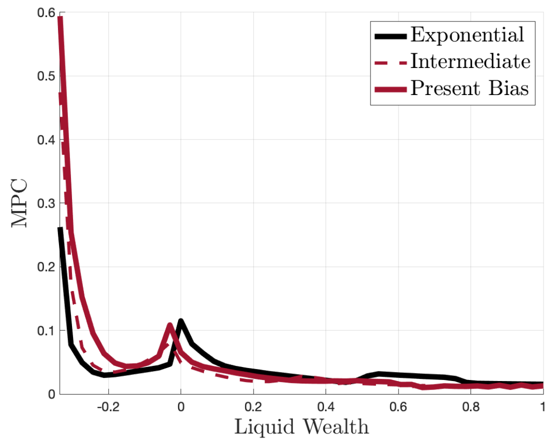
Consistent with the typical behavior in these sorts of models, the Exponential Benchmark features elevated MPCs at $b = 0$ where the interest rate jumps and at the borrowing constraint $\underline{b}$ (e.g., Kaplan and Violante, 2014; Kaplan et al., 2018). Consumption-smoothing motives imply that MPCs are low away from these constraints.

Present bias drastically increases MPCs near the borrowing constraint. This follows

[39] Equation (13) defines the MPC out of an infinitesimal increase in liquid wealth. However, tax rebates and fiscal stimulus payments increase liquid wealth discretely. The definition of the MPC can easily be extended to discrete liquidity shocks (see Appendix D.1 for details).

directly from the consumption function discontinuities at $\underline{b}$ detailed above. Present-biased households do not smooth consumption into the borrowing constraint, instead choosing a high rate of consumption all the way into $\underline{b}$.[40]

(a) Quarterly MPCs



(b) Liquid Wealth Distribution

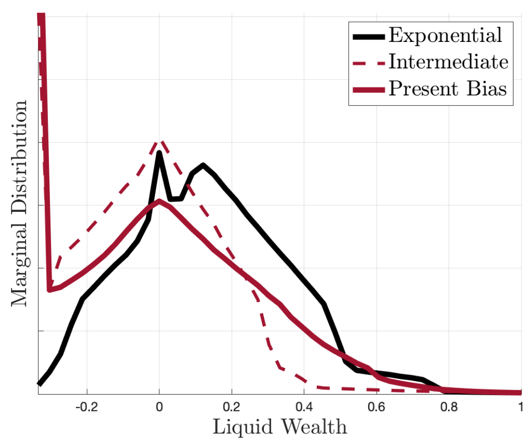


Figure 3: MPCs Over the Liquid Wealth Distribution.

Notes: The top panel presents the quarterly MPC as a function of liquid wealth, averaging over the mortgage and income dimensions. The bottom panel shows the household distribution over liquid wealth. Each panel includes all three calibration cases.

Steady State Wealth Distribution. The bottom panel of Figure 3 presents the steady state distribution of liquid wealth for the three calibrations. There are two key differences between the Exponential Benchmark and the two $\beta < 1$ cases. First, present bias generates a leftward shift in the liquid wealth distribution because present-biased households overconsume out of liquidity. Second, present bias produces a larger share of households at the

[40] Another effect of $\beta < 1$ is that there is no longer a peak in the MPC at $b = 0$. This is because the soft constraint does not typically bind in equilibrium for the $\beta < 1$ household.

borrowing constraint. 13.1% of households are constrained in the Present-Bias Benchmark, while only 0.2% of households are constrained in the Exponential Benchmark.

There are three factors that contribute to this large mass of constrained households when $\beta < 1$. First, households fail to maintain liquidity buffers. Second, the two $\beta < 1$ calibrations feature a lower value of ρ. This reduces households' willingness to refinance at $\underline{b}$ (see the Middle Income panels in Figure 1). Third, in the case of procrastination, households can be slow to refinance when they hit the constraint.

The buildup of households at the borrowing constraint is consistent with the responsiveness of debt accumulation to borrowing limits that has been documented empirically. Gross and Souleles (2002) study how borrowing responds to credit limit increases, and estimate that credit card debt increases by 10-14% of the increased limit after one year. In our model, the first-order effect of a small increase in the borrowing limit $\underline{b}$ is that constrained households accumulate additional debt by exactly the same amount, so that the overall marginal borrowing propensity simply equals the share of constrained households. Consistent with Gross and Souleles (2002), this share is 13.1% in the Present-Bias Benchmark. In the Exponential Benchmark, this share is only 0.2%. Among new credit card applicants Agarwal et al. (2018) find similarly that an increase in credit limits generates additional credit card borrowing for the majority of households.

In Figure 3 we present the liquid wealth distribution because it is key for many of our model's dynamics. Appendix Figures 11 and 12 plot the distribution of mortgage debt and the joint distribution of income, liquid wealth, and mortgage debt for all three cases.

Summary Statistics. Table 2 summarizes the model's steady state. The first row lists average consumption, which is reported relative to permanent income. Average consumption is comparable in all three cases, and is less than average income (normalized to 1) due to debt repayment. The second row lists average consumption conditional on each income state. The two $\beta < 1$ calibrations feature lower levels of consumption by low-income households. Present bias inhibits households' ability to self-insure against income fluctuations by accumulating liquid buffer stocks. This is particularly true for the Present-Bias Benchmark because liquidity-constrained households are slow to refinance.

The Present-Bias Benchmark features an average quarterly MPC of 12.5% out of a \$1,000 transfer. For the Exponential Benchmark, this MPC is 4.3%. Only the Present-Bias Benchmark comes close to empirical MPC estimates, which range from 15-25% for fiscal transfers of \$500 – \$1,000.[41] The two $\beta < 1$ calibrations also feature much larger variation in MPCs

[41] See Kaplan et al. (2018) and Kaplan and Violante (2014) for a review. Some examples of empirical MPC estimates include: Johnson et al. (2006), Parker et al. (2013), Agarwal and Qian (2014), Broda and Parker (2014), Misra and Surico (2014), and Kueng (2018). Because the model only features flow consumption, the model-implied MPC should be compared to empirical estimates of the nondurable MPC.

	Exponential	Intermediate	Present Bias
Avg. c	0.93	0.92	0.93
(y_L, y_M, y_H)	(0.84, 0.93, 1.02)	(0.83, 0.93, 1.01)	(0.81, 0.94, 1.03)
Avg. Quarterly MPC ($1,000)	4.3%	8.5%	12.5%
(y_L, y_M, y_H)	(5.2, 5.3, 2.3)	(14.8, 8.5, 2.1)	(25.8, 9.8, 2.5)
Avg. Quarterly MPC ($10,000)	4.2%	6.2%	8.7%
(y_L, y_M, y_H)	(5.2, 5.0, 2.1)	(10.6, 6.2, 1.8)	(17.5, 6.7, 2.2)
Avg. Quarterly MPX ($1,000)	13.7%	23.9%	31.7%
Avg. Quarterly MPX ($10,000)	13.2%	19.3%	26.2%
Share $b = 0$	5.9%	6.3%	4.7%
Share $b < 0$	25.8%	52.2%	46.9%
Share $b = \underline{b}$	0.2%	9.3%	13.1%

Table 2: Steady State Summary Statistics.

Notes: This table summarizes household consumption, expenditure, and saving behavior in the steady state.

based on transitory income. Low- and middle-income households have high MPCs because they compose a larger share of households on or near the borrowing constraint. Though this MPC heterogeneity still exists in the Exponential Benchmark, it is much less drastic. Time-consistent households maintain liquidity buffers optimally, so the Exponential Benchmark features far fewer constrained households.

The elevated MPC exhibited in the Present-Bias Benchmark is robust to the size of the wealth shock. The average quarterly MPC remains at 9% for a transfer of $10,000 (for more, Appendix Figure 10 reports quarterly MPCs out of transfers ranging from $1,000 to $50,000). Alternatively, to the extent that models with exponential discounting can generate large MPCs, these large MPCs decline rapidly in the size of the transfer. For example, the model of Kaplan and Violante (2014) predicts that quarterly MPCs drop from 15% to 3% as the size of the transfer increases from $500 to $5,000.[42]

When comparing the model to the data, it is important to delineate between the marginal propensity to consume (MPC) versus the marginal propensity for expenditure (MPX) (Laibson et al., 2021). Durables drive a wedge between consumption and expenditure because spending on durables does not translate into immediate consumption. The difference can be substantial empirically. For example, Parker et al. (2013) document that over 75% of spending from the 2008 fiscal stimulus was on durable goods.

To bridge the gap between consumption and expenditure, we follow the method of Laibson et al. (2021) for converting our model's predictions about MPCs into predictions about

[42]See Kueng (2018) for empirical evidence of high MPCs from large transfers. Attanasio et al. (2020) also argue that this pattern of high MPCs from large shocks is difficult to rationalize in standard two-asset models, and show that temptation preferences can account for this pattern.

MPXs.[43] Table 2 reports the model's MPX predictions. As with MPCs, the two $\beta < 1$ calibrations feature elevated quarterly MPXs that remain large even for large wealth transfers. Elevated MPXs out of large transfers is consistent with Fagereng et al. (2019a), who use Norwegian administrative data to estimate that lottery winners of amounts ranging from \$8,300 – \$150,000 spend 50% of their prize within the year of winning.

The final three rows of Table 2 summarize the accumulation of liquid wealth. In the Exponential Benchmark, only 0.2% of households are constrained, and only 26% of households hold credit card debt (compared to 60% in the SCF). The two $\beta < 1$ cases feature far less liquid wealth holding, in line with the data. In the Present-Bias Benchmark, 13% of households are constrained and 47% hold credit card debt.

5 Results: Macro Stabilization Policy with Present Bias

We now present our results for fiscal and monetary policy under present-biased time preferences. We start all policy counterfactuals from the pre-shock steady state with $r^* = 1\%$.

5.1 Fiscal Policy

We first study how present bias affects the efficacy of fiscal policy. The policy experiment is as follows. Starting from the steady state, we (unexpectedly) conduct a helicopter drop of \$1,000. To avoid unnecessarily complicating the partial equilibrium analysis, we ignore the effect of this small fiscal transfer on the government budget constraint and future taxes.[44]

Figure 4 plots the impulse response function (IRF) of aggregate consumption $C_t = \int c_t(x)g_t(x)dx$ to a fiscal transfer of \$1,000 to every household in the economy. To make this IRF easier to connect to MPC numbers, we scale the consumption response at time t by the size of the initial fiscal transfer. Table 3 gives the corresponding MPCs and MPXs for the three calibrations over different time horizons. The connection between the scaled IRF in Figure 4 and the MPC in Table 3 is simple: the MPC over a time horizon τ is the

[43] Laibson et al. (2021) microfound a simple formula to calculate the MPX over τ years:

$$MPX_\tau(x) = MPC_\tau(x) + \frac{s}{\nu + r_0} \times \frac{\partial}{\partial b}\mathbb{E}\left[c(x_\tau) \mid x_0 = x\right].$$

That is, the MPX is equal to the MPC plus a term that captures the additional durable stock the household has accumulated at time τ needed to increase the consumption flow. $r_0 = 1\%$ is the steady state interest rate, and BEA consumption data is used to calibrate durable share $s = 0.13$ and depreciation rate $\nu = 0.22$. See Laibson et al. (2021) for details.

[44] Balancing the government budget constraint is unlikely to significantly affect our results. For example, if the \$1,000 transfer were repaid by levying a constant lump-sum tax in perpetuity, this tax would be roughly \$10 per year (1% of \$1,000). Also, Ricardian equivalence does not hold in our model due to binding liquidity constraints and probabilistic death.

cumulative IRF between times $t = 0$ and $t = \tau$.[45] The results in the figure and table show clearly that the short-run potency of fiscal policy is heightened for $\beta < 1$, particularly when households procrastinate on refinancing.

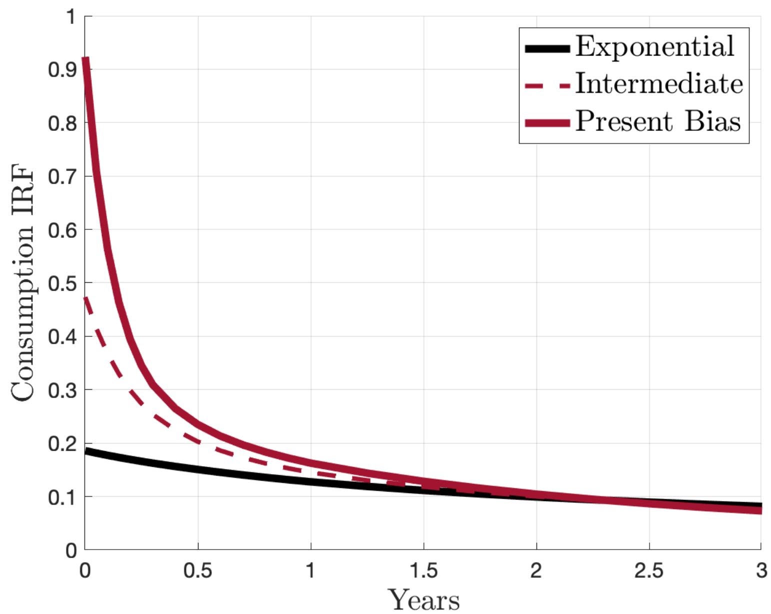


Figure 4: Consumption Response to Fiscal Policy.
Notes: This figure presents the IRF of aggregate consumption to a fiscal stimulus of $1,000 for the three calibration cases.

	Exponential	Intermediate	Present Bias
1 Year MPC	15%	22%	28%
2 Year MPC	26%	34%	41%
3 Year MPC	35%	42%	49%
1 Year MPX	22%	30%	37%
2 Year MPX	31%	39%	46%
3 Year MPX	39%	46%	53%

Table 3: Aggregate Fiscal Policy MPCs and MPXs.
Notes: This table presents aggregate MPCs and MPXs out of a $1,000 fiscal transfer.

The intuition for this fiscal policy result is provided by Figure 3. The two $\beta < 1$ calibrations feature larger MPCs at the borrowing constraint as well as a larger mass of constrained

[45]To see this, consider a small fiscal stimulus of size db. The MPC in equation (13) records cumulative consumption over a time period of length τ. In contrast, the scaled IRF records the consumption response to the small initial fiscal transfer db at a particular point in time t. Thus, denoting the scaled IRF of a household with initial state x by $IRF_t(x) = \frac{\partial}{\partial b}\mathbb{E}\left[c(x_t) \mid x_0 = x\right]$, we have $MPC_\tau(x) = \int_0^\tau IRF_t(x)dt$.

households. This dual result of large MPCs for constrained households combined with a large share of constrained households makes fiscal policy very powerful for $\beta < 1$. Table 2 and Appendix Figure 10 show that this result holds for large transfers, suggesting that present bias also allows fiscal policy to remain potent even when implemented at larger scales.

The dynamic spending response predicted by our model can be investigated using the "intertemporal MPCs" of Auclert et al. (2018). Auclert et al. (2018) argue that this dynamic spending response is important for characterizing the general equilibrium propagation of fiscal policy shocks. Using empirical evidence from Fagereng et al. (2019a), Auclert et al. (2018) show that MPXs remain elevated in the years following an initial shock. The spending response in our Present-Bias Benchmark is consistent with this finding: the MPX in year 1 is 37%, the MPX from year 1 to year 2 is 9%, and the MPX from year 2 to year 3 is 7%.[46]

Fiscal Policy Implementation: Liquidity. In response to the 2007-08 Financial Crisis, policymakers utilized a mixture of liquid and illiquid fiscal transfers (e.g., checks in the mail versus mortgage principal reductions). Because the elevated MPCs in the $\beta < 1$ cases are driven by liquidity-constrained households, our model implies that the liquidity of fiscal transfers is critical for generating a large consumption response.

To illustrate this result, Figure 5 compares the consumption response following a liquid fiscal transfer to a mortgage reduction of the same magnitude. The mortgage reduction has much less short-run potency for the $\beta < 1$ cases. This is because the liquidity-constrained households who drive the consumption response are unable to immediately consume out of illiquid home equity. The extreme sensitivity of the consumption response to the liquidity of the transfer is consistent with the empirical findings of Ganong and Noel (2020), who use variation in mortgage modification programs following the financial crisis to document that liquidity is the critical driver of short-term household consumption decisions.[47] Alternatively, the consumption response in the Exponential Benchmark is far less sensitive to the liquidity of the transfer, inconsistent with the Ganong and Noel (2020) evidence.

5.2 Monetary Policy

Next we study the impact of stimulative monetary policy in our economy with present-biased households. Starting from the steady state, interest rates are cut from 1% to 0%. Approximately 70% of households find themselves in a refinancing region following the rate cut. By encouraging households to refinance, an important feature of this interest rate cut is

[46]The dynamic spending response should be compared to a stripped-down version of our Present-Bias Benchmark without income uncertainty and borrowing constraints, which yields a linear consumption policy function and an annual MPX of less than 2%. See Appendix D.2 for details.

[47]See also Eberly and Krishnamurthy (2014) for a discussion.

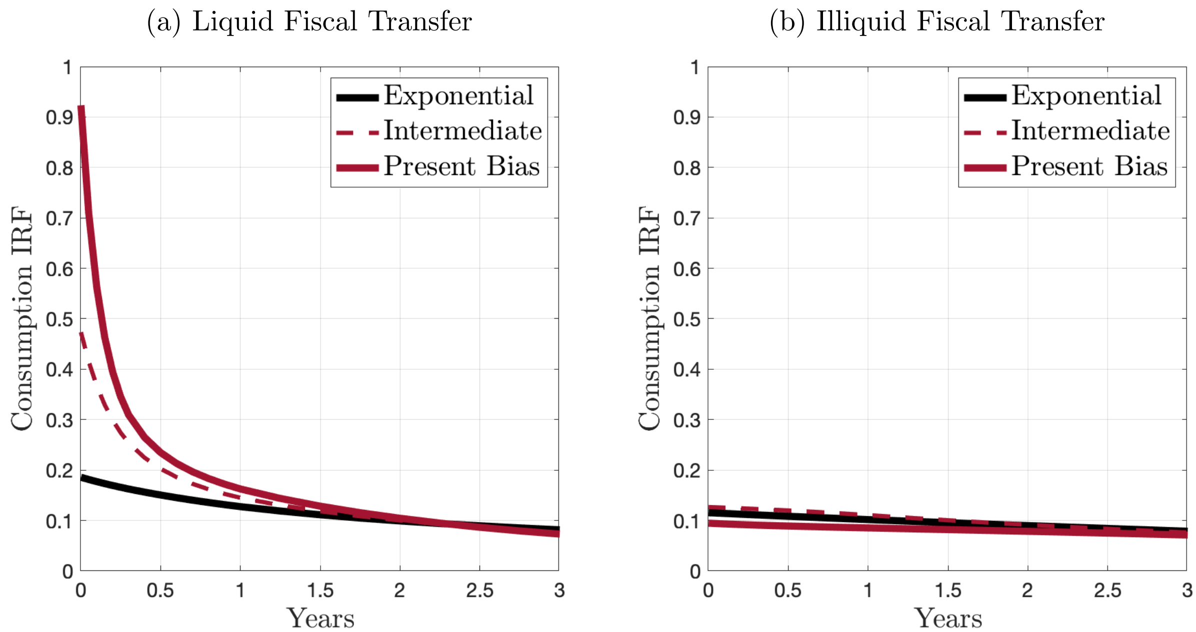


Figure 5: Liquidity of Fiscal Policy.
Notes: The left panel reproduces the benchmark fiscal policy analysis of Figure 4. The right panel plots the IRF of aggregate consumption to a \$1,000 mortgage principal reduction.

that it also provides households with an opportunity to extract home equity. Indeed, roughly 75% of households who refinance engage in a cash-out refinance.[48] Details of the refinancing decision are presented in Appendix E.3.

Figure 6 illustrates the consumption response to this 1% rate cut. The Exponential Benchmark features a 3.8% increase in consumption on impact, which then decays slowly over longer horizons. The rate cut induces a wave of cash-out refinances on impact, and this extracted home equity is steadily spent down over time.

Next we introduce present bias. Starting with the Intermediate Case with no procrastination, monetary policy is more than 33% more effective on impact. The intuition for this result is the same as for fiscal policy. The rate cut incentivizes households, especially those who are constrained, to extract equity from their home. In this way, *the refinancing channel of monetary policy imitates the liquidity-injection features of fiscal policy.* Since present-biased households overconsume out of liquid wealth, this wave of home-equity extractions produces a large consumption boom. Though fiscal and monetary policy differ in terms of where the liquidity injection comes from – fiscal policy gives households liquidity directly while monetary policy incentivizes households to extract liquidity from their home – both are powerful because they provide liquidity to constrained households with large MPCs.

In addition to generating a larger consumption response on impact, the Intermediate

[48] Chen et al. (2020) estimate that approximately 70% of mortgage refinancings from 1993 – 2010 involved an equity withdrawal.

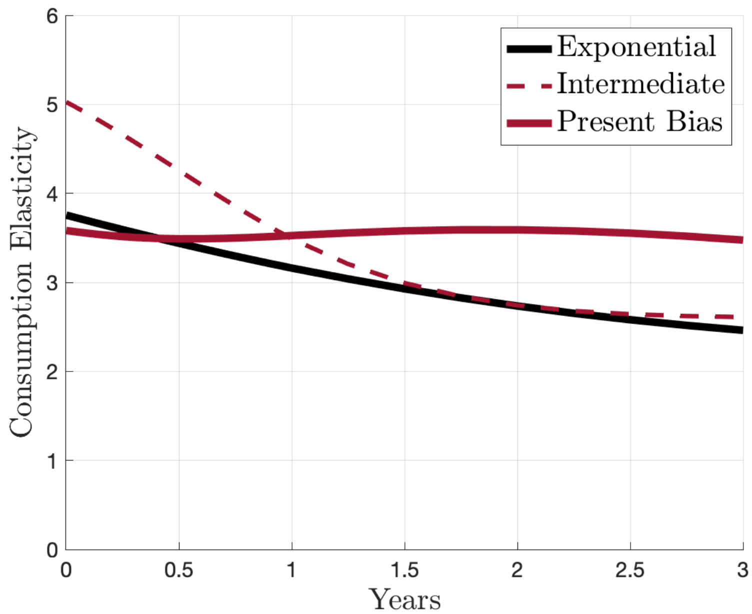


Figure 6: Consumption Response to Monetary Policy.

Notes: For the three calibration cases, this figure plots the percent change in aggregate consumption following an interest rate cut from 1% to 0%.

Case features a faster burn-out of monetary policy over time. In both the Exponential Benchmark and the Intermediate Case, households withdraw equity from their homes at the time of the interest rate shock. In the Exponential Benchmark this liquidity is consumed optimally, while in the Intermediate Case it is overconsumed. This results in a larger, but shorter lived, consumption boom.

Turning to the Present-Bias Benchmark, procrastination reshapes the timing of the consumption response to monetary policy. The Present-Bias Benchmark features a smaller effect on impact than the Intermediate Case. But, the Present-Bias Benchmark also exhibits essentially no decay in potency over the three-year period that we study. With procrastination, the effect of monetary policy is spread out over time rather than concentrated at the time of the rate cut. This produces a less drastic, but longer lived, economic stimulus.

Bringing all three cases together, constrained households with high MPCs compose the dry powder that is ignited by the cash-out channel of monetary policy. The effect of $\beta < 1$ is to create a larger stock of dry powder. However, the speed at which this dry powder is ignited depends on procrastination.

This intuition also suggests that the effect of procrastination itself depends on present bias. We confirm this intuition in Appendix Figure 14. Appendix Figure 14 compares the consumption response in the Exponential Benchmark to an alternate $\beta = 1$ economy augmented with refinancing procrastination.[49] Augmenting the Exponential Benchmark with procrastination has little effect on the consumption response to monetary policy. Consumption is much less sensitive to procrastination in the Exponential Benchmark because time-consistent households hold a larger stock of liquid wealth. This allows them to buffer consumption while waiting to refinance.

A noteworthy feature of the Present-Bias Benchmark is that the consumption response to monetary policy in Figure 6 is mildly hump-shaped, reaching its peak after around 2 years. Appendix Figure 16 shows that this hump shape can also be more pronounced if the procrastination duration is shorter. A hump-shaped response of aggregate consumption to monetary policy shocks is a common finding in the literature estimating such impulse response functions using time-series data (e.g., Rotemberg and Woodford, 1997; Christiano et al., 2005). Present bias with procrastination thus has the potential to qualitatively generate this empirical finding.[50]

There is also an emerging literature showing that the refinancing channel of monetary policy is sensitive to the time-path of mortgage interest rates (Berger et al., 2018b; Eichenbaum et al., 2018). Similar patterns arise in our model, but are not the focus of our analysis. We highlight a different form of sensitivity — sensitivity to procrastination. Procrastination means that households are slow to adjust on their refinancing margin, prolonging the pass-through of monetary policy to household consumption.

Consumption Response Decomposition: Three Channels. Monetary policy affects household consumption through three channels. First, there is the standard direct effect on liquid wealth — the change in interest rate r_t affects the household's return on b_t.[51] Second, the interest rate cut gives households the option to refinance into a lower-rate mortgage. Third, households can extract home equity when refinancing their mortgage.

We decompose the initial consumption response to monetary policy into its three components. First, we isolate the direct effect on liquid wealth by shutting down households' ability to refinance. We do this by making κ^{refi} arbitrarily large. Second, we reintroduce the ability for households to conduct a rate refinance, but keep the cash-out channel shut down. To do this, we modify the refinancing budget constraint (equation (5)) as follows:

$$b' - m' = b_t - m_t - \kappa^{refi}, \text{ subject to } m' \in [0, m_t + \kappa^{refi}] \text{ and } b' \geq \underline{b}. \tag{5'}$$

[49] Given the effort cost structure in Assumptions 1 and 2, one can generate procrastination by setting $\beta = 0.999$.

[50] Auclert et al. (2020) alternatively propose "sticky expectations" as a potential source of such hump-shaped impulse response functions.

[51] In particular, the direct effect on liquid wealth includes the usual income and substitution effects, which are the focus of most single-asset models of monetary policy.

Constraint (5′) means that households cannot increase their liquid wealth balances by refinancing; i.e., $b' \leq b_t$.[52] Third, we reintroduce the cash-out channel to get back to the benchmark results shown in Figure 6.

This decomposition is presented in Table 4. Each cell presents the on-impact consumption elasticity in the modified model.[53] Roughly one quarter of the total consumption response can be attributed to the liquid wealth channel, and another quarter can be attributed to the rate refinancing channel. In all three cases, the majority of the total consumption response comes from the cash-out channel of monetary policy.

	Exponential	Intermediate	Present Bias
Step 1. No Refis	0.79 (21%)	0.89 (18%)	0.83 (23%)
Step 2. No Cash-Outs	1.74 (46%)	2.08 (41%)	1.81 (50%)
Step 3. Full Response	3.76 (100%)	5.03 (100%)	3.58 (100%)

Table 4: Consumption Response Decomposition.

Notes: This table decomposes the channels through with monetary policy produces a consumption response (on impact). The first row presents the consumption elasticity when households are not allowed to refinance. The second row allows for rate refinances, but not home-equity extractions. The third row presents the consumption response in the full model. Parentheses indicate the share of the total consumption response.

5.3 Summary

In an economy with present-biased households, fiscal policy is a powerful and robust tool for macroeconomic stabilization. Monetary policy is a powerful yet sensitive policy tool. Present bias creates both high MPCs and a large share of constrained households. These households are quick to spend down liquidity, making fiscal policy a high-powered tool for generating a short-run consumption boom. Present bias increases the potency of monetary policy for a similar reason: rate cuts produce cash-out refinances, which mirror the liquidity-injection features of fiscal policy. Though powerful, the refinancing channel of monetary policy is sensitive to refinancing inertia. Procrastination increases the lag between rate cuts and refinancing, generating a milder but longer-lived stimulus.

Figure 7 summarizes the effect of present bias on fiscal and monetary policy. Figure 7 plots our fiscal and monetary policy experiments scaled by the consumption response in the Exponential Benchmark. For fiscal policy, $\beta < 1$ robustly generates a larger short-run consumption boom. For monetary policy, the effect of $\beta < 1$ is sensitive to procrastination. The Intermediate Case features a larger, but short-lived, consumption boom. The Present-Bias

[52] Specification (5′) does allow households to roll the fixed cost of refinancing into their new mortgage. This is necessary to allow households at $\underline{b}$ to refinance.

[53] In all three steps we use the pre-shock distribution of households from the full model. This prevents the distribution of households from changing as we change households' access to refinancing technology.

(a) Fiscal Policy (b) Monetary Policy

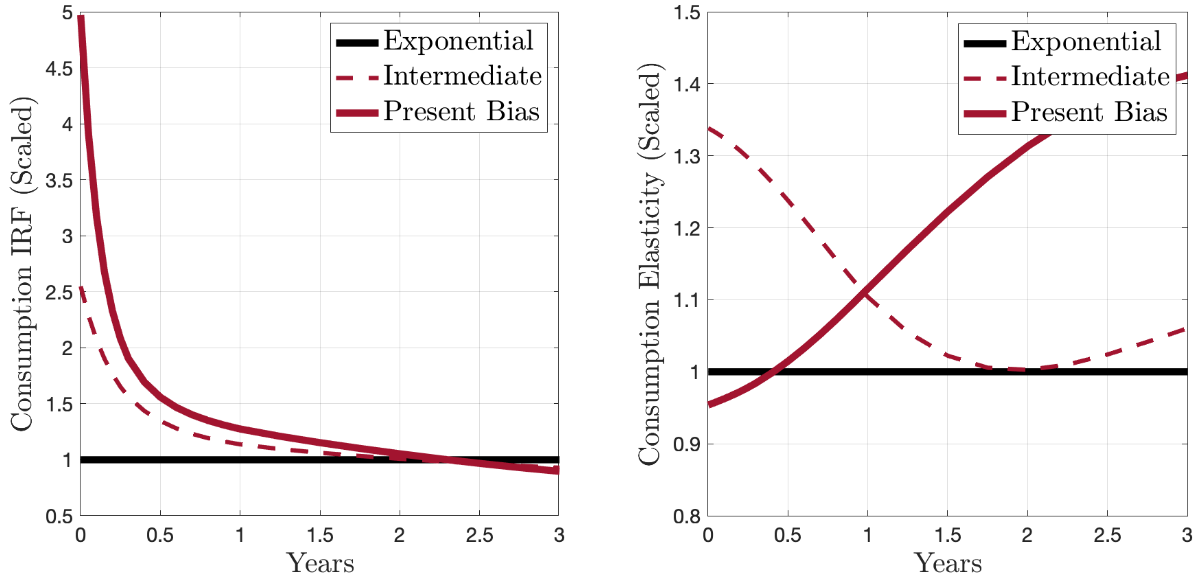


Figure 7: Fiscal and Monetary Policy, Scaled to Exponential Benchmark.
Notes: The left panel presents our fiscal policy analysis, and the right panel presents our monetary policy analysis. All consumption responses are scaled to the Exponential Benchmark.

and Exponential Benchmarks feature a similar consumption response on impact. However, the Present-Bias Benchmark produces a much larger long-run response as the dry powder of constrained households is slowly ignited.

5.4 The Distributional Effects of Policy

We end by leveraging the heterogeneous-agent structure of the model to explore how present bias affects the distributional consequences of fiscal and monetary policy.[54]

The top row of Figure 8 breaks down the consumption response to fiscal policy for our two benchmark cases. Each panel plots the consumption response to fiscal stimulus on impact as a function of pre-shock consumption. In the Exponential Benchmark (left) the consumption response is relatively evenly spread across the consumption distribution. In the Present-Bias Benchmark (right) this is not the case — the lowest consumption households experience a drastic consumption boom from fiscal policy. These households are borrowing-constrained, and sharply increase consumption following the liquidity shock.

The bottom row of Figure 8 breaks down the consumption response to monetary policy on impact. In the Exponential Benchmark, the largest consumption response comes from low-consumption households. These households are near $\underline{b}$, and implement a cash-out refinance

[54] Wolf (2021) conducts a similar comparison of the distributional effects of monetary and fiscal policy in a full general equilibrium model but without present bias and a mortgage refinancing channel.

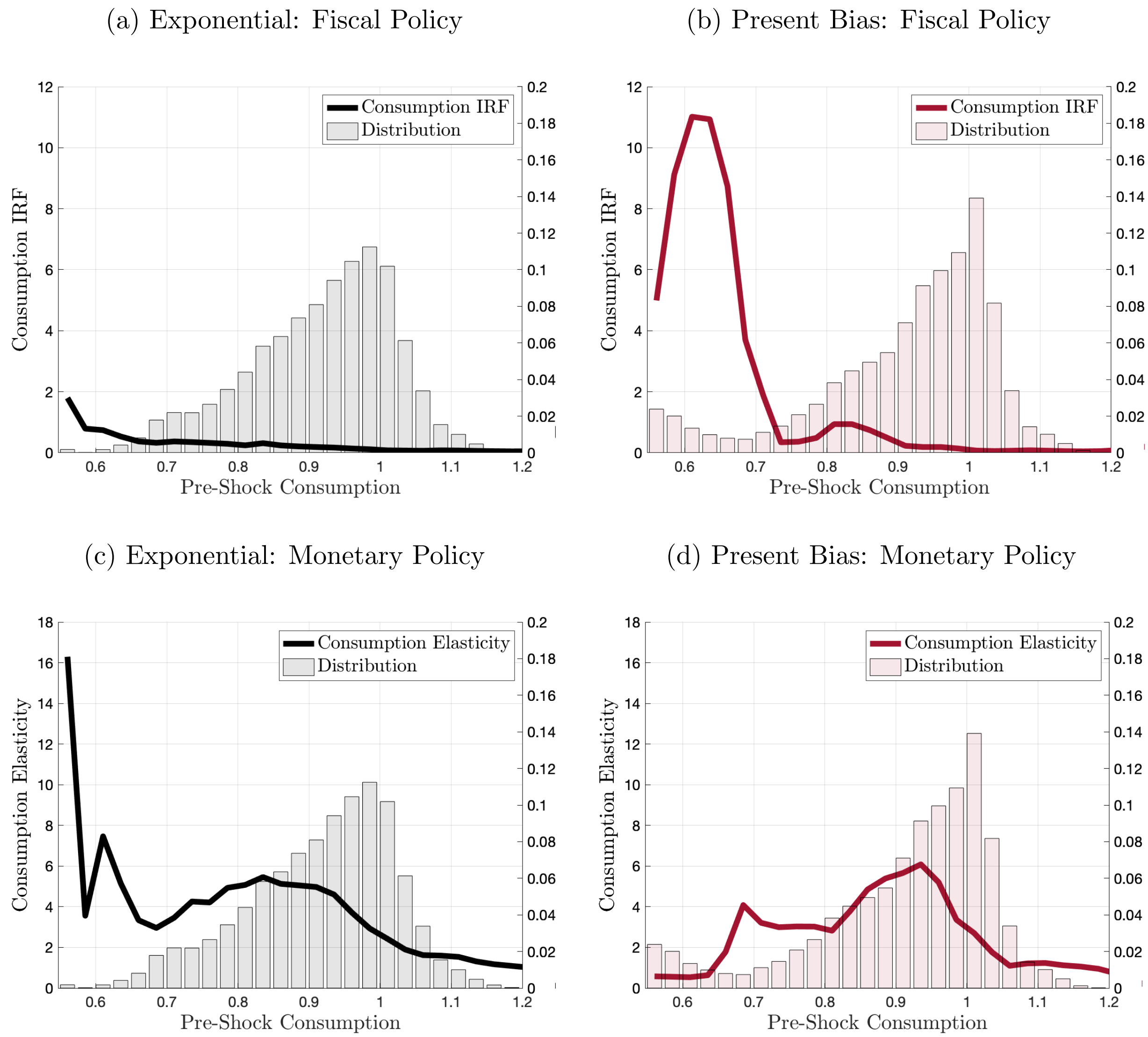


Figure 8: Heterogeneity Analysis.

Notes: This figure plots the on-impact consumption response to fiscal (top row) and monetary (bottom row) policy as a function of households' pre-shock consumption. The solid line plots the consumption response, and the bars show the distribution of households over pre-shock consumption.

following the rate cut. Thus, in the Exponential Benchmark the refinancing channel of monetary policy endogenously targets itself to constrained households.

Alternatively, in the Present-Bias Benchmark the low-consumption households respond very little to monetary policy on impact. The largest response now comes from households with intermediate levels of pre-shock consumption. Low-consumption households are constrained on impact, and because they procrastinate on refinancing they cannot immediately adjust consumption. Households with intermediate consumption are not liquidity constrained on impact. These households will typically be in either a refinancing region following the rate cut (in which case they expect to refinance in the next instant), or near a refinancing region (in which case they expect to refinance soon). In both cases, consumption smoothing implies that these households will increase consumption today in expectation of the cash-out refinance that they plan to conduct in the near future. This ability to smooth consumption relies on pre-existing liquidity at the time of the monetary policy shock, which households at $\underline{b}$ do not have.

The key takeaway from Figure 8 is that present bias reverses the distributional consequences of fiscal versus monetary policy. In the Exponential Benchmark, monetary policy is an effective way to stimulate the consumption of low-consumption households: a cut to interest rates allows low-consumption households to refinance. In the Present-Bias Benchmark, procrastination implies that monetary policy no longer stimulates the short-run consumption of constrained households. Instead, fiscal policy is highly effective at increasing the short-run consumption of low-consumption households.

It is important to emphasize that distributional considerations also become more important under present bias. While less than 1% of households are constrained in the Exponential Benchmark, over 13% of households are constrained in the Present-Bias Benchmark. The impact of stabilization policy on constrained households affects a much larger share of the population in the Present-Bias Benchmark.

6 Adding Aggregate House Price and Income Shocks

Macroeconomic stabilization policy seldom operates in isolation and instead responds to shocks hitting the economy. In particular, expansionary monetary and fiscal policy are often used in recessions. Recessionary shocks, by definition, correspond to a temporary decline in aggregate income. Recessions can also coincide with declining house prices. This section examines the ways in which shocks to house prices and aggregate income affect our results.

6.1 House Price Shocks

Present bias amplifies monetary policy by producing a consumption boom driven by home-equity extractions. However, this cash-out channel of monetary policy is limited to households with enough home equity to actually conduct a cash-out refinance. This makes monetary policy sensitive to house price shocks, which can quickly create or destroy home equity.

Understanding the effect of house price shocks on macroeconomic policy is particularly important when considering the three most recent recessions: the COVID-19 Recession, the Great Recession, and the Early 2000s Recession. Home prices collapsed during the Great Recession, but boomed throughout the Early 2000s Recession and the COVID-19 Recession (to date).

Our baseline analysis in Section 5 corresponds to the case where home prices are stable before the cut to interest rates. To examine the effect of house price shocks we exogenously shock the home value h by $\pm$ 25%.[55] The negative 25% shock corresponds to the Great Recession. The positive 25% shock corresponds to the early 2000s, where house prices boomed while the Federal Reserve adopted a multi-year path of low interest rates. Second, policymakers immediately respond to this house price shock with either monetary or fiscal policy. As in Section 5, the monetary policy experiment is a rate cut from 1% to 0%, and the fiscal policy experiment is a $1,000 liquid transfer.

Figure 9 plots the consumption response to monetary policy after a negative (left panel) or positive (right panel) 25% shock to house prices. The solid curves plot the consumption response to monetary policy in the shocked economy. For reference, the transparent lines mark the baseline case in Figure 6. Though the magnitude of the consumption response is sensitive to house price shocks, our main result that present bias amplifies the consumption response to monetary policy holds in both cases.

(a) -25% House Price Shock

(b) +25% House Price Shock

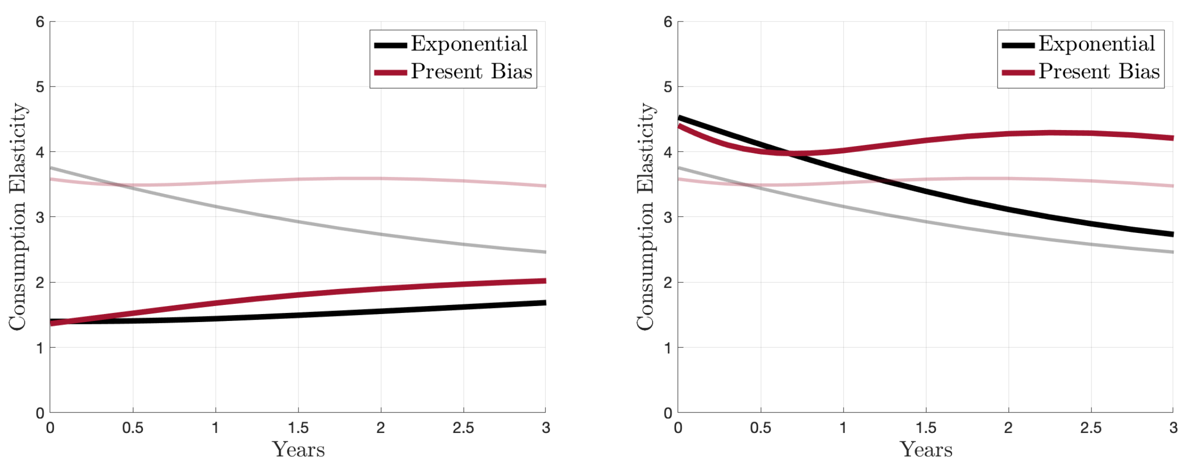


Figure 9: Monetary Policy and House Price Shocks.
Notes: This figure plots the consumption response to an interest rate cut that immediately follows a house price shock of -25% (left) or +25% (right). The transparent lines plot the baseline case in Figure 6, and are included for reference.

The left panel of Figure 9 shows that monetary policy is significantly weakened by the collapse in house prices. The negative shock wipes out home equity and prevents many homeowners from refinancing. This result is consistent with recent research documenting that negative house price shocks undermined monetary policy following the Great Recession (e.g., Beraja et al., 2019).

[55]The economy starts in the "steady state" before the shock to h. For simplicity we assume that the shock is permanent. However, we only study the short-run consumption response to monetary and fiscal policy.

The right panel of Figure 9 plots the positive 25% shock case. Now, the consumption boom generated by the rate cut is even larger than in the baseline case. The positive shock generates additional home equity, strengthening the cash-out channel of monetary policy. This is consistent with the boom in home-equity extractions that was observed in the mid-2000s (Khandani et al., 2013; Bhutta and Keys, 2016).

It is also important to explore whether house price shocks affect fiscal policy. For both positive and negative house price shocks, we find that present bias continues to strongly amplify the consumption response to fiscal policy.[56] Details are in Appendix E.5.

6.2 Income Shocks

We also evaluate the effect of recessionary income shocks on monetary and fiscal policy. We generate a temporary 5% fall in aggregate income by shifting a share of high-income households to the middle-income state, and a share of middle-income households to the low-income state.[57] Policymakers immediately respond to this recessionary income shock with either monetary or fiscal policy.

The recessionary income shock leads to an immediate decline in consumption of roughly 1%. However, the subsequent consumption response to monetary and fiscal policy is almost identical to the baseline results in Section 5. This is because liquidity, not income, is the key driver of the consumption response to these policies. Appendix E.5 contains further details.

7 Extensions

7.1 A Call to ARMs?

In order to reflect the typical features of the U.S. mortgage market our paper studies macroeconomic stabilization policy under the assumption that households have fixed-rate mortgages (FRMs). Since the 2007-08 Financial Crisis, many economists have argued that downwardly flexible mortgages, such as adjustable-rate mortgages (ARMs), improve macroeconomic stability (e.g., Eberly and Krishnamurthy, 2014; Andersen et al., 2020; Campbell et al., 2020; Guren et al., 2021). If the monetary authority cuts interest rates in a recession, ARM payments automatically adjust downward, thereby increasing households' disposable income. This creates a fast and direct transmission of monetary policy to household balance sheets.

[56] For example, in the negative 25% shock case the one-year MPC is 18% in the Exponential Benchmark, compared to 28% in the Present-Bias Benchmark. In the positive 25% shock case the one-year MPC is 10% in the Exponential Benchmark, compared to 26% in the Present-Bias Benchmark.

[57] We shift 9.5 percentage points of high-income households to middle income, and 9.5 percentage points of middle-income households to low income. The share of households across low, middle, and high income goes from 31%, 39%, 31%, respectively, to 40%, 39%, 21%.

Section 5.2 shows that refinancing procrastination slows down the transmission of monetary policy. A natural policy question is whether such procrastination implies that downwardly flexible mortgages would improve the potency of monetary policy in our model with present bias.

To study this question, we re-solve our Present-Bias Benchmark under the assumption that all households have ARMs instead of FRMs. The model from Section 2 remains the same, except that mortgage rate r_t^m automatically adjusts with interest rate r_t. We also recalibrate the mortgage wedge from 1.7% to 0.9%.[58] This corresponds to the average difference between a 5/1 hybrid ARM and the 10-year treasury yield from 2015 – 2017.

In our monetary policy experiment with ARMs we cut r_t from 1% down to -1%. This doubles the magnitude of the rate cut from our earlier FRM analysis, where interest rates were reduced from 1% to 0%. This change ensures comparability across the two experiments, since ARMs are more sensitive to monetary policy than long-duration FRMs.[59]

We find that the consumption response to monetary policy is almost identical with ARMs versus FRMs (see Appendix Figure 15 for details). This result highlights that there is a tradeoff between ARMs and FRMs that arises when households are present biased. On the one hand, ARMs produce a fast pass-through of monetary policy that applies to all mortgage holders. This is particularly important for constrained households who procrastinate on refinancing. On the other hand, ARMs reduce the liquidity injection features of monetary policy because ARMs imply that households no longer need to refinance when the interest rate is cut. Present bias generates a powerful cash-out channel of monetary policy, but this channel is stifled by ARMs. Overall, the stimulative effect of ARMs accrues quickly and to all households, but is small. The stimulative effect of FRMs accrues slowly and only to households who plan to refinance, but is large. These two effects are of similar magnitude in our model.

An important factor explaining these offsetting effects is the large size difference between ARM payment adjustments versus home-equity extractions. Recall that the home value is calibrated to 3.1 times permanent income, and the average LTV ratio is 0.54. With ARMs, a 2% reduction in mortgage rates is equivalent to a $2\% \times 0.54 \times 3.1 = 3\%$ increase in income for the average household (roughly \$3,000 per year). Alternatively, with FRMs the typical cash-out is about 30% of permanent income (\$30,000) – an order of magnitude larger than the typical ARM payment reduction – and present-biased households have large MPCs out of these liquidity injections. Such large cash-outs are consistent with the data. The average cash-out amount from 1999–2010 was \$40,000 (Bhutta and Keys, 2016), and there is little

[58] FRMs are typically more expensive than ARMs because FRMs lead to lower payments if interest rates rise, and come with the option to refinance if rates fall. The borrower has to pay ex-ante for this insurance.

[59] Empirically there is roughly a 50% pass-through from monetary policy to the 30-year mortgage interest rate. See Gertler and Karadi (2015), Gilchrist et al. (2015), and Eichenbaum et al. (2018) for details.

evidence that the majority of these home-equity extractions are kept as savings (Greenspan and Kennedy, 2008; Bhutta and Keys, 2016). However, ARMs prevent this large stock of dry powder from ever being ignited.

While we do not find that ARMs increase the power of monetary policy, we note that our model is too stylized to make rigorous quantitative claims. Our analysis also assumes that house prices are fixed. Negative house price shocks, such as those observed following the financial crisis, can significantly reduce the cash-out channel of FRMs (see Section 6.1). Our results nevertheless highlight a new tradeoff between FRMs and ARMs that policymakers should be aware of when considering different mortgage contract designs. Our results also suggest that monetary policy is most powerful if mortgage contracts feature a fast pass-through (like ARMs) while simultaneously allowing for cash-outs (like FRMs). Appendix Figure 16 shows just how powerful monetary policy can be in a FRM environment if policymakers are able to reduce procrastination concurrently with a monetary expansion.

In addition to being stylized this section ignores important welfare considerations. For example, FRMs produce a consumption boom by encouraging overconsumption out of home equity. Monetary policy also appears to be more equitable under ARMs than FRMs. In Section 5.4 we showed that low-consumption households procrastinate on refinancing a FRM, whereas ARMs provide immediate payment relief to low-consumption households.

7.2 Discussion: Implications of Present Bias for General Equilibrium Effects of Macroeconomic Policy

As we have already discussed, our model is set in partial equilibrium because abstracting from general equilibrium considerations allows for a richer, and more straightforward, investigation of the household problem. This raises the question of how present bias would affect the transmission of monetary and fiscal policy in a full general equilibrium analysis. Here we briefly discuss this question through the lens of the literature on Heterogeneous Agent New Keynesian (HANK) models. That is, we ask what the effect of present bias would be on the consumption response to monetary and fiscal policy in a general equilibrium version of the model with nominal rigidities.[60]

In HANK models, macroeconomic stabilization policy can trigger a number of different

[60] One could also imagine studying the impact of present bias on the consumption response to macroeconomic policy in models without nominal rigidities, and this may overturn our result that present bias amplifies this response. For example, in a model with a classical dichotomy, changes in nominal interest rates would have no effect on real consumer spending regardless of whether the economy features present bias. Similarly, one may be able to construct a general equilibrium version of our model in which an extreme form of Ricardian equivalence holds so that fiscal stimulus has no effect on consumer spending, again regardless of present bias. We view such exercises as less interesting and instead discuss environments in which policy affects consumer spending also in the absence of present bias.

indirect general equilibrium effects, particularly effects working through household labor income, asset prices, and returns (see e.g. Werning, 2015; Kaplan et al., 2018; Auclert, 2019; Alves et al., 2020; Slacalek et al., 2020). The size of these indirect effects depends on the size of these variables' movements as well as households' responsiveness to these changes, e.g. MPCs (and MPXs) out of labor income and asset price changes. In heterogeneous-agent models with idiosyncratic income risk and borrowing constraints of the type analyzed here these indirect effects can be important because such models often generate sizable MPCs (e.g., Kaplan et al., 2018).

As we have shown above, present bias increases both households' average MPC and the direct consumption effect of an interest rate cut. The likely implications for the transmission of monetary and fiscal policy in a full general equilibrium analysis are as follows.

Fiscal Policy. We conjecture that, also in a general equilibrium HANK version of our model, present bias would continue to amplify households' spending response to fiscal policy. This follows from a simple "Keynesian cross" logic which takes as its starting point that the most potent general equilibrium effect triggered by fiscal policy is likely the one working through households' labor incomes: a fiscal transfer increases aggregate consumption demand (the impulse or direct effect); in equilibrium, firms hire more which increases households' labor incomes and leads to additional spending (the multiplier or indirect effect).[61] The key ingredient determining the size of both this impulse and multiplier are households' MPCs, which increase with present bias. Present bias therefore likely amplifies not only the direct effects but also the indirect general equilibrium effects of fiscal policy.

Monetary Policy. We conjecture that the situation is similar for monetary policy, namely that present bias would increase not only direct but also indirect effects and therefore the overall consumption response. Just like fiscal policy, monetary policy triggers indirect effects working through labor income and present bias would amplify these via higher MPCs. Monetary policy can also trigger indirect effects working through asset prices and returns (Gornemann et al., 2016; Kaplan et al., 2018; Alves et al., 2020; Slacalek et al., 2020).[62] However, since present bias does not significantly affect MPCs out of liquid wealth for high-liquidity households (see Figure 3), nor MPCs out of illiquid wealth (see Figure 5), it is natural to conjecture that present bias does little to impact the indirect effects working through asset prices and returns. Taken together, this discussion suggests that, in a HANK-

[61] For modern macro versions of this mechanism, see for example Auclert et al. (2018) and Wolf (2019).

[62] Since we study the effect of monetary policy on consumption at relatively high frequencies, we are mostly interested in asset price changes at those same frequencies. In this regard, empirical evidence usually points to interest rate cuts as increasing stock prices (e.g., Bernanke and Kuttner, 2005; Gürkaynak et al., 2005). There is also evidence that loose monetary policy increases house prices (e.g., Jordà et al., 2015), but this mechanism seems to operate at a lower frequency than we study.

version of our model, present bias would continue to amplify the effects of monetary policy once general equilibrium effects are taken into account.

Fully evaluating the impact of present bias on the economy's response to monetary and fiscal policy in a general equilibrium model is an important task for future work.

8 Conclusion and Policy Discussion

The goal of this paper is twofold. First, we show that present bias improves the model's ability to replicate a variety of empirical patterns exhibited in household consumption-saving behavior. Second, we use our model to understand how present bias alters the household balance-sheet channels of macroeconomic policy.

Our results on fiscal policy yield a clear conclusion: liquid wealth transfers to constrained households produce a powerful consumption response. Present bias creates both a large share of constrained households and high MPCs for these households. These high MPCs imply that even large fiscal transfers (e.g., \$10,000) will be consumed quickly.

The model's monetary policy predictions are more nuanced. Because the refinancing channel of monetary policy replicates the liquidity-injection features of fiscal policy, monetary policy is powerful for $\beta < 1$. However, the timing of this refinancing channel is affected by refinancing inertia. Unlike fiscal policy, it can take time for the full effects of monetary policy to reveal themselves. The large share of constrained households who intend to refinance – but are slow to do so – implies that the "dry powder" of home equity is slow to ignite. This also suggests that policies aimed at reducing refinancing inertia can increase the short-run impact of monetary policy (Andersen et al., 2020).

Present bias also reshapes the distributional consequences of fiscal and monetary policy. When $\beta = 1$, monetary policy endogenously targets constrained households because these households will endogenously select into (cash-out) refinancing. Intuitively, by cutting interest rates the policymaker naturally helps households who borrow. When $\beta < 1$, procrastination overturns this result. Constrained households no longer rush to refinance, and instead persist in their low-consumption state. Fiscal policy becomes the relevant tool for quickly improving the consumption of constrained households.

A digression on the targeting of fiscal policy is warranted. Our model shows that households near $\underline{b}$ are the ideal targets for fiscal policy. Though a household's liquid-wealth state is likely unobservable to policymakers, our model suggests that income is highly correlated with liquid wealth. Indeed, Table 2 shows how large MPCs are for low- and middle-income households. Here, too, a warning is needed. All households in our model are ex-ante identical, and we model transitory income fluctuations around a constant permanent level. While policymakers may not be able to observe permanent income, unemployment is an observable state that will include a large share of households earning less than their permanent income level. This practical implication accords with the empirical results of Ganong and Noel (2019), who find that consumption is highly sensitive to unemployment insurance benefits.

We conclude by repeating a number of limitations of our analysis. First, we do not model general equilibrium forces and touched upon this issue only briefly in Section 7.2. As discussed there, we view our results as inputs into richer general equilibrium models. Second, our model abstracts from many important macroeconomic dimensions. We focus on a subset of the population, homeowners. We do not model endogenous responses of the financial sector nor do we model businesses, both of which will experience stress during recessions. Our model does not feature a government, and there are no future tax liabilities associated with our fiscal policy experiments. Our discussion on the timing of fiscal and monetary policy abstracts from policy lags which, in practice, are a critical difference between the speed of fiscal versus monetary policy. Third, even in partial equilibrium the household side of our model is highly stylized. We ignore detailed lifecycle dynamics as well as many non-housing assets like stock market wealth. The model also omits mortgage default, which may interact with both monetary policy and present bias in interesting ways. Fourth, we do not study the welfare consequences of fiscal and monetary policy. All of these considerations are likely fruitful areas for future research.

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Appendix

A Calibration Details

A.1 Summary of External Calibration

	Description	Value	Target / Source
Preferences			
γ	Risk Aversion	2	Literature
ϕ	Procrastination Decay Rate	$-\log(0.5)$	Andersen et al. (2020)
Income			
y_t	Transitory Income	$\{0.75, 0.98, 1.28\}$	Guerrieri and Lorenzoni (2017)
$\mathbf{A}^y$	Income Transition Matrix	(see text)	Guerrieri and Lorenzoni (2017)
Interest Rates			
r_t	Short Rate	$\{-1\%, 0\%, 1\%, 2\%\}$	10-Year TIPS
$\mathbf{A}^r$	Short Rate Transition Matrix	(see text)	10-Year TIPS
ω^{cc}	Credit Card Wedge	10.3%	Credit Card - 10-Yr Treasury Spread
ω^m	Mortgage Wedge	1.7%	30-Yr FRM - 10-Yr Treasury Spread
Assets and Liabilities			
h	House Value	3.1	2016 SCF
θ	Max LTV	0.8	Greenwald (2018)
ξ	Mortgage Paydown	0.035	20 Year Half-Life
κ^{prepay}	Prepayment Fixed Cost	0.002	Numerical Stability
κ^{refi}	Refinancing Fixed Cost	0.05	FRB Documentation
$\underline{b}$	Credit Limit	$-\frac{1}{3}$	2016 SCF
Other Structural Assumptions			
λ^F	Rate of Forced Adjustment	$\frac{1}{15}$	2016 CPS Avg. Moving Rate
λ^R	Retirement Rate	$\frac{1}{30}$	Average Working Life
y^R	Retirement Fixed Income	y_L	Retirement Replacement Rate
-	Birth Distribution	$m_0 = \theta h, b_0 \sim U(0, y_L)$	Lifecycle Dynamics

Table 5: Externally Calibrated Parameters.

Notes: This table presents the model's externally calibrated parameters. See Section 4.1 for details.

A.2 SCF Details

Many of our calibrated parameters rely on data from the 2016 SCF. To construct a sample of households that is consistent with our model we impose the following data filters. The head of house must be in the labor force and aged 25-66. The household must own a home (with weakly positive home equity), possess a credit card, earn no income from Social

Security nor retirement accounts, and have after-tax permanent income between the 1st and 99th percentile. On this sample, we then condition on households with a home value to permanent income ratio between the 25th and 75th percentile.

All of our variables are scaled relative to permanent income. Following Kennickell (1995), Kennickell and Lusardi (2004), and Fulford (2015) we use the SCF's question about "normal income" to measure each household's permanent income.[63] Though this is an imperfect proxy for the household's permanent income, it has the benefit of being both straightforward and respecting the household's information set. We adjust each household's normal income for 2015 federal taxes, and deduct an additional 5% for state taxes.

We use the 2016 SCF to estimate six moments that are used in our calibration: (i) permanent income; (ii) average home value to permanent income; (iii) average LTV; (iv) average credit card debt to permanent income; (v) share of households with revolving credit card debt; and (vi) average credit limit to permanent income. Moments (ii) – (v) are reported in the main text. The average after-tax permanent income for our sample of homeowners is $95,918. The average credit limit to permanent income is 0.36.

A.3 Estimation and Discretization of Ornstein-Uhlenbeck Processes

To calibrate our income and interest rate processes, we assume that these processes are discretized versions of continuous-time Ornstein-Uhlenbeck (OU) processes.

Consider a generic mean-zero OU process $u(t) = \int_0^t e^{-\kappa(t-s)}\sigma dZ_s$. Process $u(t)$ has the conditional distribution $u(t+\tau)|u(t) = \mathcal{N}\left(u(t)e^{-\kappa\tau}, \frac{\sigma^2}{2\kappa}(1-e^{-2\kappa\tau})\right)$.

Assume that $u(t)$ is only observed in discrete snapshots every Δ years. Let $d_s = u(s\Delta)$ denote the s'th snapshot of process $u(t)$. The discrete process d_s can be modeled as an AR(1) process:

$$\begin{aligned} d_{s+1} &= \rho d_s + \sigma_d \varepsilon_{s+1}, \text{ where} \\ \rho &= e^{-\kappa\Delta} \\ \sigma_d^2 &= \frac{\sigma^2}{2\kappa}(1-e^{-2\kappa\Delta}). \end{aligned}$$

Given any discrete-time AR(1) estimate, we can use the above formulas to back out the parameters of the underlying OU process: κ and σ. We discretize the OU process using standard finite difference methods. For details, see the numerical Appendix of Achdou et al. (2021).

[63] SCF respondents are asked whether or not their 2015 income was normal. If not, they are asked to report what their total income would be if it had been normal.

B Naive Present Bias: Passing to Continuous Time

Here we present a heuristic derivation of naive IG preferences as the continuous-time limit of a model where some of the decisions are made discretely. This heuristic approach is designed to capture the intuition of the more rigorous derivation in Harris and Laibson (2013). We begin by assuming a constant effort cost, as in Sections 2.1 and 2.2. The full setup with a stochastic effort cost, as introduced in Section 2.3, is presented in Appendix B.3.

B.1 Naive IG Current-Value Function

Assume that the current self lives for a discrete length of time, denoted Δ. After this time has elapsed, starting with the next self, time progresses continuously again.[64] Since the naive present-biased household incorrectly perceives that all future selves will discount exponentially, continuation-value function $v(x)$ characterizes the equilibrium starting with the next self at time Δ. The current self discounts all future selves by β, so the current-value function for the naive present-biased household is given by:

$$\begin{aligned} w(x) &= \max\left\{ \max_c \; u(c)\Delta + \beta e^{-\rho\Delta}\mathbb{E}[v(x_\Delta)|x],\; w^*(x) - \bar{\varepsilon} \right\} \quad \text{with} \\ w^*(x) &= \max\left\{ w^{prepay}(x), w^{refi}(x) \right\} \\ w^{prepay}(x) &= \max_{b',m'} w(b', m', y, r^m, r) \quad \text{ s.t. prepayment constraint (4) holds} \\ w^{refi}(x) &= \max_{b',m'} w(b', m', y, r + \omega^m, r) \quad \text{ s.t. refinancing constraint (5) holds} \end{aligned} \tag{14}$$

and where x_Δ denotes the vector of household states after time interval Δ has elapsed, for example $b_\Delta = b + (y + rb + \omega^{cc}b^- - (r^m + \xi)m - c)\Delta$.

Equation (14) captures the consumption/adjustment decisions made by the current self. In the left branch of the first line the household does not adjust, and chooses consumption rate c over the next Δ units of time to maximize the current-value function. In the right branch of the first line the household pays effort cost $\bar{\varepsilon}$ and fixed monetary cost κ^i to discretely adjust its mortgage. Importantly, this discrete-time value function is written such that there is no delay to refinancing (i.e., the current self benefits from refinancing).[65] Though this is unrealistic – there are time delays in refinancing – we write the Bellman equation in this

[64] This mixed discrete- and continuous-time setup is of course slightly non-standard. Alternatively, we could have assumed that future selves also make decisions in discrete time, as done in Laibson and Maxted (2020). In this case the continuation-value function $v(x)$ would be the discrete-time analogue of the continuous-time $v(x)$ that we use below.

[65] To see how the value function is written in this way, note that refinancing gives the current self the current-value function of w^*. As the first line of equation (14) shows, this value function consists of an undiscounted utility flow earned for the current self, $u(c)\Delta$.

way to emphasize that our results do not rely on assumptions about temporal delays.

Discrete-time Bellman equation (14) can be used to derive the current-value function in continuous time. Taking the time-step Δ to its continuous-time limit, we see that the term $u(c)\Delta$ drops out of the current-value function, leaving:

$$w(x) = \max\left\{\beta v(x),\ w^*(x) - \bar{\varepsilon}\right\}.$$

This recovers equation (9) in the main text.

B.2 Continuous Control: Consumption (Proof of Lemma 1)

We now derive the continuous-time first-order condition for consumption stated in Lemma 1. As shown in equation (14), the household makes a consumption choice in every period. For the consumption decision, equation (14) implies that consumption is given by the following first-order condition:[66]

$$u'(c(x)) = \beta e^{-\rho\Delta}\ \frac{\partial}{\partial b}\mathbb{E}[v(x_\Delta)|x].$$

Taking $\Delta \to 0$ yields

$$u'(c(x)) = \beta\frac{\partial v(x)}{\partial b},$$

which is equation (11) in Lemma 1. This derivation continues to hold in the full setup with a stochastic effort cost presented in Appendix B.3 below.

B.3 Full Setup with a Stochastic Effort Cost (Section 2.3)

Here we briefly spell out the full set of equations for the generalization with a stochastic effort cost that evolves according to the two-state process in Assumption 1. In what follows, we will denote value and policy functions in the normal high-cost state by the same functions as in the baseline model with a constant effort cost, e.g. $v(x)$ or $\mathfrak{R}(x)$. Alternatively, we will denote the corresponding value and policy functions in the temporary low-cost state with underlines, e.g. $\underline{v}(x)$ or $\underline{\mathfrak{R}}(x)$.

We first show how to generalize equation (8′), the HJBQVI equation for the value function

[66] We ignore difficulties such as the kink in the budget constraint when taking this first-order condition.

$v(x)$ of a $\beta = 1$ household:

$$
\begin{aligned}
\rho v(x) = \max \Bigg\{ \max_c \Big\{ u(c) + & \frac{\partial v(x)}{\partial b}(y + rb + \omega^{cc} b^- - (r^m + \xi)m - c) \Big\} \\
& - \frac{\partial v(x)}{\partial m}(\xi m) \\
& + \sum_{y' \neq y} \lambda^{y \to y'} \big[v(b, m, y', r^m, r) - v(b, m, y, r^m, r) \big] \\
& + \sum_{r' \neq r} \lambda^{r \to r'} \big[v(b, m, y, r^m, r') - v(b, m, y, r^m, r) \big] \\
& + \lambda^R \big[v^R(x) - v(x) \big] \\
& + \lambda^F \big[v^*(x) - (v(x) - \bar{\varepsilon}) \big] \\
& + \phi \big[\underline{v}(x) - v(x) \big], \\
& \rho(v^*(x) - \bar{\varepsilon}) \Bigg\}.
\end{aligned}
\tag{8''}
$$

Relative to (8′), there is a new entry $\phi\big[\underline{v}(x) - v(x)\big]$. Parameter ϕ is the arrival rate of the low-effort-cost state, and $\underline{v}(x)$ is the household's value in this state. This value is given by

$$\underline{v}(x) = \max\{v(x), v^*(x) - \underline{\varepsilon}\}. \tag{15}$$

Intuitively, since the low-cost state only lasts for an instant (Assumption 1), the household either takes advantage of refinancing at the lower effort cost $\underline{\varepsilon}$ or it loses the opportunity in the next instant in which case its value reverts back to $v(x)$.

Instead of asserting equation (15) and justifying it with economic reasoning as we just did, we can also derive it from a full HJBQVI equation for $\underline{v}(x)$ that is symmetric to (8″) and in which the effort cost switches from $\underline{\varepsilon}$ to $\bar{\varepsilon}$ at a Poisson rate $\underline{\phi}$; then take $\underline{\phi} \to \infty$ (Assumption 1).

We next show how to generalize (9), the equation for the current-value function $w(x)$:

$$
\begin{aligned}
w(x) &= \max \Big\{ \beta v(x),\ w^*(x) - \bar{\varepsilon} \Big\} \quad \text{and} \\
\underline{w}(x) &= \max \Big\{ \beta v(x),\ w^*(x) - \underline{\varepsilon} \Big\} \quad \text{with} \\
w^*(x) &= \max \big\{ w^{prepay}(x), w^{refi}(x) \big\} \\
w^{prepay}(x) &= \max_{b',m'}\ w(b', m', y, r^m, r) \quad \text{s.t. prepayment constraint (4) holds} \\
w^{refi}(x) &= \max_{b',m'}\ w(b', m', y, r + \omega^m, r) \quad \text{s.t. refinancing constraint (5) holds}
\end{aligned}
\tag{16}
$$

Relative to (9), there is a new line $\underline{w}(x) = \max\left\{\beta v(x),\ w^*(x) - \underline{\varepsilon}\right\}$ that captures the current-value of a household that has the opportunity to refinance at the low-effort-cost $\underline{\varepsilon}$.

Like in Appendix B.1, the current-value function in (16) can be derived from a setup in which the current self lives for a discrete length of time Δ:

$$
\begin{aligned}
w(x) &= \max\left\{\max_c\ u(c)\Delta + \beta e^{-\rho\Delta}\left[e^{-\phi\Delta}\mathbb{E}[v(x_\Delta)|x] + \left(1 - e^{-\phi\Delta}\right)\mathbb{E}[\underline{v}(x_\Delta)|x]\right],\ w^*(x) - \bar{\varepsilon}\right\},\\
\underline{w}(x) &= \max\left\{\max_c\ u(c)\Delta + \beta e^{-\rho\Delta}\left[e^{-\underline{\phi}\Delta}\mathbb{E}[\underline{v}(x_\Delta)|x] + \left(1 - e^{-\underline{\phi}\Delta}\right)\mathbb{E}[v(x_\Delta)|x]\right],\ \underline{w}^*(x) - \underline{\varepsilon}\right\},\\
w^*(x) &= \max\left\{w^{prepay}(x), w^{refi}(x)\right\}\\
w^{prepay}(x) &= \max_{b',m'} w(b', m', y, r^m, r) \quad \text{s.t. prepayment constraint (4) holds}\\
w^{refi}(x) &= \max_{b',m'} w(b', m', y, r + \omega^m, r) \quad \text{s.t. refinancing constraint (5) holds}\\
\underline{w}^*(x) &= \max\left\{\underline{w}^{prepay}(x), \underline{w}^{refi}(x)\right\}\\
\underline{w}^{prepay}(x) &= \max_{b',m'} \underline{w}(b', m', y, r^m, r) \quad \text{s.t. prepayment constraint (4) holds}\\
\underline{w}^{refi}(x) &= \max_{b',m'} \underline{w}(b', m', y, r + \omega^m, r) \quad \text{s.t. refinancing constraint (5) holds}
\end{aligned}
$$

where ϕ and $\underline{\phi}$ denote the Poisson switching rates between the two effort-cost states. As stated in Assumption 1 we assume that $\underline{\phi} \to \infty$. Therefore $e^{-\underline{\phi}\Delta} \to 0$ and

$$
\begin{aligned}
w(x) &= \max\left\{\max_c\ u(c)\Delta + \beta e^{-\rho\Delta}\left[e^{-\phi\Delta}\mathbb{E}[v(x_\Delta)|x] + \left(1 - e^{-\phi\Delta}\right)\mathbb{E}[\underline{v}(x_\Delta)|x]\right],\ w^*(x) - \bar{\varepsilon}\right\},\\
\underline{w}(x) &= \max\left\{\max_c\ u(c)\Delta + \beta e^{-\rho\Delta}\mathbb{E}[v(x_\Delta)|x],\ \underline{w}^*(x) - \underline{\varepsilon}\right\},\\
w^*(x) &= \max\left\{w^{prepay}(x), w^{refi}(x)\right\}\\
w^{prepay}(x) &= \max_{b',m'} w(b', m', y, r^m, r) \quad \text{s.t. prepayment constraint (4) holds}\\
w^{refi}(x) &= \max_{b',m'} w(b', m', y, r + \omega^m, r) \quad \text{s.t. refinancing constraint (5) holds}\\
\underline{w}^*(x) &= \max\left\{\underline{w}^{prepay}(x), \underline{w}^{refi}(x)\right\}\\
\underline{w}^{prepay}(x) &= \max_{b',m'} \underline{w}(b', m', y, r^m, r) \quad \text{s.t. prepayment constraint (4) holds}\\
\underline{w}^{refi}(x) &= \max_{b',m'} \underline{w}(b', m', y, r + \omega^m, r) \quad \text{s.t. refinancing constraint (5) holds}
\end{aligned}
$$

Finally, we take the limit as $\Delta \to 0$. Using the property that $\underline{w}^*(x) = w^*(x)$ in the limit as $\Delta \to 0$ – which one can see by inspection since the left branch of the first line converges to the left branch of the second line – we recover equation (16).

C Proofs

C.1 Proof of Corollary 1

Recall that, with naiveté, the perceived continuation-value function of a $\beta < 1$ household equals the value function of an exponential $\beta = 1$ household and solves (8′′). Assume that the household does not refinance at time t so that the perceived continuation-value function $v(x_t)$ is characterized by a standard HJB equation. This HJB equation is given by the left branch of (8′′), which we write here as

$$\rho v(x) = \max_c \ u(c) + \frac{\partial v(x)}{\partial b}(y + rb + \omega^{cc} b^- - (r^m + \xi)m - c) + (\mathcal{B}v)(x) \tag{17}$$

where the operator $(\mathcal{B}v)(x)$ is short-hand notation for lines two to seven of (8′′). Recall that we use hat-notation to denote the policy functions that naive households perceive for future selves, and denote by $\widehat{c}(x)$ and $\widehat{s}(x) = (y + rb + \omega^{cc} b^- - (r^m + \xi)m - \widehat{c}(x))$ the corresponding perceived consumption and liquid saving policy functions. In contrast, denote by $c(x)$ (from Proposition 1) and $s(x) = (y + rb + \omega^{cc} b^- - (r^m + \xi)m - c(x))$ the *actual* policy functions.

The following observation is important in the proof below: the HJB equation for the perceived continuation-value function (17) features the *perceived* policy functions $\widehat{c}(x), \widehat{s}(x)$, rather than the *actual* policy functions. But what determines the evolution of liquid wealth b are the *actual* policy functions.

Differentiate (17) with respect to b and use the envelope theorem:

$$(\rho - r(b))\frac{\partial v(x)}{\partial b} = \frac{\partial^2 v(x)}{\partial b^2}\widehat{s}(x) + \frac{\partial}{\partial b}(\mathcal{B}v)(x). \tag{18}$$

Define the marginal continuation-value of wealth $\eta(x) \equiv \frac{\partial v(x)}{\partial b}$. From (18) it satisfies

$$(\rho - r(b))\, \eta(x) = \frac{\partial \eta(x)}{\partial b}\widehat{s}(x) + (\mathcal{B}\eta)(x). \tag{19}$$

If $\beta = 1$, from Itô's formula, the right-hand side of (19) also governs the expected change in the marginal value of wealth: $\mathbb{E}_t[d\eta(x_t)] = \left[\frac{\partial \eta(x_t)}{\partial b}\widehat{s}(x_t) + (\mathcal{B}\eta)(x_t)\right] dt$. But with $\beta < 1$ this is no longer true: the evolution of b is governed by the *actual* drift $s(x)$ rather than the *perceived* drift $\widehat{s}(x)$ and so

$$\mathbb{E}_t[d\eta(x_t)] = \left[\frac{\partial \eta(x_t)}{\partial b} s(x_t) + (\mathcal{B}\eta)(x_t)\right] dt. \tag{20}$$

Therefore, evaluating (19) along a particular trajectory x_t, we have

$$(\rho - r(b_t))\,\eta(x_t) = \frac{1}{dt}\mathbb{E}_t[d\eta(x_t)] - \frac{\partial \eta(x_t)}{\partial b}(s(x_t) - \widehat{s}(x_t)).$$

Rearranging

$$\begin{aligned}
\frac{1}{dt}\mathbb{E}_t[d\eta(x_t)] &= (\rho - r(b_t))\,\eta(x_t) + \frac{\partial \eta(x_t)}{\partial b}(s(x_t) - \widehat{s}(x_t)) \\
&= (\rho - r(b_t))\,\eta(x_t) + \frac{\partial \eta(x_t)}{\partial b}(\widehat{c}(x_t) - c(x_t)) \\
&= (\rho - r(b_t))\,\eta(x_t) + \frac{\partial \eta(x_t)}{\partial b}\left(\beta^{\frac{1}{\gamma}} - 1\right) c(x_t)
\end{aligned}$$

Finally, recalling that $\eta(x) \equiv \frac{\partial v(x)}{\partial b}$, the first-order condition is $u'(c(x)) = \beta\eta(x)$ and therefore

$$\begin{aligned}
\frac{1}{dt}\mathbb{E}_t[du'(c(x_t))] &= (\rho - r(b_t))\,u'(c(x_t)) + \frac{\partial u'(c(x_t))}{\partial b}\left(\beta^{\frac{1}{\gamma}} - 1\right) c(x_t) \\
&= (\rho - r(b_t))\,u'(c(x_t)) - u''(c(x_t))c(x_t)\left(1 - \beta^{\frac{1}{\gamma}}\right)\frac{\partial c(x_t)}{\partial b} \\
&= \left[\rho + \gamma\left(1 - \beta^{\frac{1}{\gamma}}\right)\frac{\partial c(x_t)}{\partial b} - r(b_t)\right] u'(c(x_t)),
\end{aligned}$$

where going from the second line to the third line uses that, with CRRA utility, the coefficient of relative risk aversion is $\gamma = \frac{-u''(c(x_t))c(x_t)}{u'(c(x_t))}$. Dividing by $u'(c(x_t))$, we have (12). ■

C.2 Proof of Proposition 2

When proving Proposition 2, we refer to Appendix B.3 which spells out the full set of equations for the model with a stochastic effort cost satisfying Assumption 1.

We also note that clauses 1, 2a, and 2b do not rely on the infinite Poisson switching rate used in Assumption 1. The purpose of Assumption 1 is to create the sorts of deadlines that incentivize present-biased agents to complete effortful tasks (clause 2c).

C.2.1 Proof of Proposition 2, Clause 1

The proof of clause 1 follows from equation (16). Equation (16) shows that we can rewrite w^{prepay} and w^{refi} as:

$$\begin{aligned}
w^{prepay}(x) &= \max_{b',m'}\ \beta v(b', m', y, r^m, r) \quad \text{s.t. prepayment constraint (4) holds} \\
w^{refi}(x) &= \max_{b',m'}\ \beta v(b', m', y, r + \omega^m, r) \quad \text{s.t. refinancing constraint (5) holds}
\end{aligned}$$

These are exactly the same formulas as for v^{prepay} and v^{refi} in (7), except that there is an additional β discount factor. Since the additional β discount factor has no effect on the optimal choice of (b', m'), we recover clause 1 of Proposition 2 — the choice of (b', m') is independent of β. ■

Since $v^*(x) = \max\left\{v^{prepay}(x), v^{refi}(x)\right\}$ and $w^*(x) = \max\left\{w^{prepay}(x), w^{refi}(x)\right\}$, the above proof also implies that:

$$w^*(x) = \beta v^*(x). \tag{21}$$

This property will be used in the proof of clause 2 of Proposition 2.

C.2.2 Proof of Proposition 2, Clause 2

For clause 2a, when $\beta = 1$ the assumption that $\bar{\varepsilon}$ and $\underline{\varepsilon}$ are vanishingly small (Assumption 2) implies that $v(x)$ is arbitrarily close to $\underline{v}(x)$. Accordingly, policy function $\mathfrak{R}(x)$ converges pointwise to $\underline{\mathfrak{R}}(x)$ as the effort cost vanishes.

To prove clause 2b (procrastination when $\beta < 1$ and $\varepsilon = \bar{\varepsilon}$), consider the self in control at point x in the state space. Recall from (16) that the current-value function is given by $w(x) = \max\{\beta v(x), w^*(x) - \bar{\varepsilon}\}$. Therefore the current self will not adjust their mortgage when the value from not adjusting, $\beta v(x)$, is larger than the value from adjusting, $w^*(x) - \bar{\varepsilon}$.

The value of not adjusting is given by

$$\beta v(x) \geq \beta(v^*(x) - \bar{\varepsilon}), \tag{22}$$

where the inequality $v(x) \geq v^*(x) - \bar{\varepsilon}$ follows directly from equation (8″).

Alternatively, adjusting requires the household to incur the effort cost $\bar{\varepsilon}$ in the current period and the value of adjusting is given by

$$w^*(x) - \bar{\varepsilon} = \beta v^*(x) - \bar{\varepsilon}, \tag{23}$$

where the equality follows from equation (21).

Comparing the two alternatives (22) and (23) shows that the $\beta < 1$ household will always prefer to procrastinate whenever $\varepsilon_t = \bar{\varepsilon}$, since

$$\beta(v^*(x) - \bar{\varepsilon}) > \beta v^*(x) - \bar{\varepsilon}.$$

Procrastination enables the effort cost $\bar{\varepsilon}$ to be discounted by β, while there is at most an infinitesimal cost to delaying refinancing for an instant.

To prove clause 2c (no procrastination when $\beta < 1$ and $\varepsilon = \underline{\varepsilon}$), consider the self in control at point x in the state space. Following the second line of equation (16), it will be (weakly) optimal for the current self to adjust their mortgage if and only if:

$$w^*(x) - \underline{\varepsilon} \geq \beta v(x).$$

Above, the left-hand side is the current-value from refinancing at effort cost $\underline{\varepsilon}$, and the right-hand side is the current-value from not refinancing and having the effort cost reset immediately to $\bar{\varepsilon}$. Since $w^*(x) = \beta v^*(x)$ (see equation (21)), this can be rewritten as

$$\beta v^*(x) - \underline{\varepsilon} \geq \beta v(x). \tag{24}$$

First, consider the case in which the next self is expected to adjust the mortgage if the current self procrastinates.[67] Since the next self is expected to have $\beta = 1$, this means $\widehat{\mathfrak{R}}(x) > 0$. In this case, equation ($8''$) implies that $v(x) = v^*(x) - \bar{\varepsilon}$. Plugging this into (24) shows that the current self will adjust their mortgage whenever $\beta v^*(x) - \underline{\varepsilon} \geq \beta v^*(x) - \beta\bar{\varepsilon}$ or

$$\underline{\varepsilon} \leq \beta\bar{\varepsilon},$$

which is satisfied because Assumption 1 imposes that $\underline{\varepsilon} < \beta\bar{\varepsilon}$. Intuitively, this says that the current self will adjust their mortgage now if the cost of doing so, $\underline{\varepsilon}$, is less than the discounted cost of adjusting next period, $\beta\bar{\varepsilon}$. Thus, if $\widehat{\mathfrak{R}}(x) > 0$ then $\underline{\mathfrak{R}}(x) = \widehat{\mathfrak{R}}(x)$, meaning that the household does not procrastinate.

Next, consider the case in which a $\beta = 1$ household would not refinance at point x, even in the low-effort-cost state $\varepsilon_t = \underline{\varepsilon}$, i.e., $\widehat{\underline{\mathfrak{R}}}(x) = 0$. In that case, equation (15) implies that $v(x) \geq v^*(x) - \underline{\varepsilon}$. Multiplying by β, this also implies that $\beta v(x) \geq \beta v^*(x) - \beta\underline{\varepsilon}$, and therefore

$$\beta v(x) > \beta v^*(x) - \underline{\varepsilon}.$$

Comparing this to equation (24) shows that it will not be optimal for the naive present-biased household to refinance. This is intuitive — if it is not optimal for a $\beta = 1$ household to refinance, there is no reason for it to be optimal for a naive $\beta < 1$ household to refinance. Thus, if $\widehat{\underline{\mathfrak{R}}}(x) = 0$ then $\underline{\mathfrak{R}}(x) = \widehat{\underline{\mathfrak{R}}}(x)$.

Tying these two cases together, we have shown that:

1. If $\widehat{\mathfrak{R}}(x) > 0$ then $\underline{\mathfrak{R}}(x) = \widehat{\mathfrak{R}}(x)$
2. If $\widehat{\underline{\mathfrak{R}}}(x) = 0$ then $\underline{\mathfrak{R}}(x) = \widehat{\underline{\mathfrak{R}}}(x)$

Since clause 2a of Proposition 2 implies that $\widehat{\underline{\mathfrak{R}}}(x)$ converges pointwise to $\widehat{\mathfrak{R}}(x)$ as the effort

[67] Note that the next self will face the high-effort-cost $\bar{\varepsilon}$ if the current self procrastinates when $\varepsilon_t = \underline{\varepsilon}$.

cost vanishes, the first bullet above can be rewritten as: if $\widehat{\underline{\mathfrak{R}}}(x) > 0$ then $\underline{\mathfrak{R}}(x)$ converges pointwise to $\widehat{\underline{\mathfrak{R}}}(x)$. This completes the proof of clause 2c of Proposition 2. ∎

D Supplements to Sections 4 and 5

D.1 MPCs and MPXs out of Discrete Wealth Shocks

In Section 4 the MPC and the MPX are defined over infinitesimal wealth shocks. Following Achdou et al. (2021), this section extends these definitions to discrete wealth shocks.

Let $C_\tau(x) = \mathbb{E}\left[\int_0^\tau c(x_t)dt \mid x_0 = x\right]$ denote total expected consumption from time 0 to time τ. Recall that $x = (b, m, y, r^m, r)$. Let $x + \chi$ be shorthand for the vector $(b + \chi, m, y, r^m, r)$, i.e. $x + \chi$ is point x plus a liquid wealth shock of size χ.

For a discrete liquidity shock of size χ the MPC is defined as:

$$MPC_\tau^\chi(x) = \frac{C_\tau(x+\chi) - C_\tau(x)}{\chi}.$$

The MPX is defined as (see Laibson et al. (2021) for details):

$$MPX_\tau^\chi(x) = MPC_\tau^\chi(x) + \frac{s}{\nu + r_0}\left(\frac{\mathbb{E}\left[c(x_\tau) \mid x_0 = x + \chi\right] - \mathbb{E}\left[c(x_\tau) \mid x_0 = x\right]}{\chi}\right).$$

Total consumption $C_\tau(x)$, which is used in the MPC calculation, can be calculated numerically using a Feynman-Kac formula (see Lemma 2 of Achdou et al. (2021) for details). To calculate the MPX we also need to solve for the expected consumption rate at time τ, $\mathbb{E}\left[c(x_\tau)|x_0 = x\right]$. Again, a Feynman-Kac formula can be used to solve for this directly.[68] Numerically, we solve the Feynman-Kac formula for the sample path $r_t = 1\%$ for all t (i.e., no aggregate interest rate shocks) since these calculations are conducted in the steady state.

D.2 Naive Present Bias in Continuous Time: A Cake-Eating Model

A key motivation for this paper is to understand how present bias interacts with the complexities of household balance sheets. For comparison, this section eliminates those complexities and calculates the effect of naive present bias in a simplified "Eat-the-Cake" model.

In this textbook model, households are infinitely lived, have deterministic income $\bar{y}$, and have access to a single liquid asset with return r. Relative to the model in the main text, this stripped-down model eliminates income uncertainty, hard and soft borrowing constraints, multiple assets of varying liquidity, monetary policy, and retirement.

[68] The Feynman-Kac formula for $C_\tau(x)$ is provided in Achdou et al. (2021). The Feynman-Kac formula for $\mathbb{E}\left[c(x_\tau)|x_0 = x\right]$ is specified slightly differently. Here, $\mathbb{E}\left[c(x_\tau)|x_0 = x\right]$ is given by $\Gamma(x, 0)$, where $\Gamma(x, 0)$ satisfies the PDE $0 = (\mathcal{A}\Gamma)(x, t)$ subject to the terminal condition $\Gamma(x, \tau) = c(x)$.

We normalize $\bar{y} = 0$. In this simple model, the consumption function is given by:

$$c(b) = \beta^{-\frac{1}{\gamma}} \left(r - \frac{r - \rho}{\gamma} \right) \times b.$$

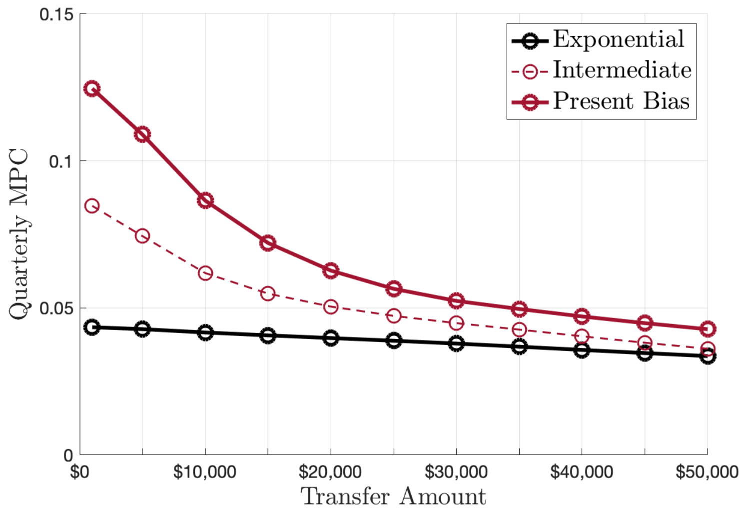
The term in parentheses is the consumption function for an exponential household (see e.g. Fagereng et al. (2019b)), and multiplicative factor $\beta^{-\frac{1}{\gamma}}$ adjusts for present bias (see Proposition 1).

For our Present-Bias Benchmark calibration the consumption rate is approximately 1%, which corresponds to an annual MPC of 1% and an annual MPX of less than 2%. These should be compared to our main model, which produces an annual MPC of 28% and an annual MPX of 37%. Our Exponential Benchmark calibration also produces a similar annual MPC and MPX in this stripped-down model, but should be compared to an annual MPC and MPX in the main model of only 15% and 22%, respectively. This highlights that present bias can amplify the effect of certain balance sheet complexities on household consumption-saving decisions.

E Additional Results

E.1 Model Solution Details: MPCs

(a) Quarterly MPCs Across Transfer Amounts



(b) Present-Bias Benchmark: MPCs over Liquid Wealth

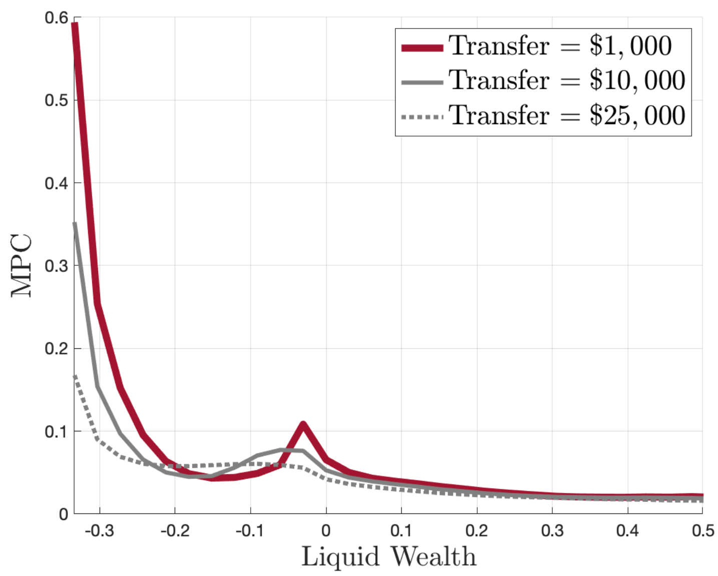


Figure 10: MPCs Across Transfer Amounts.

Notes: For the three calibration cases, the top panel plots quarterly MPCs out of transfers ranging from \$1,000 to \$50,000. The bottom panel replicates the MPC analysis in Figure 3 for the Present-Bias Benchmark calibration across transfer amounts of \$1,000 (benchmark), \$10,000, and \$25,000.

E.2 Model Solution Details: Steady State Distributions

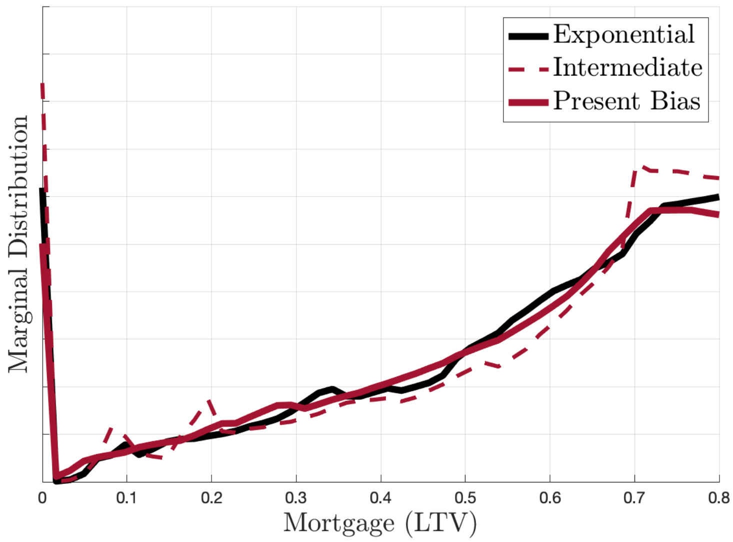


Figure 11: LTV Distribution.

Notes: This figure shows the steady state distribution of households over the LTV ratio.

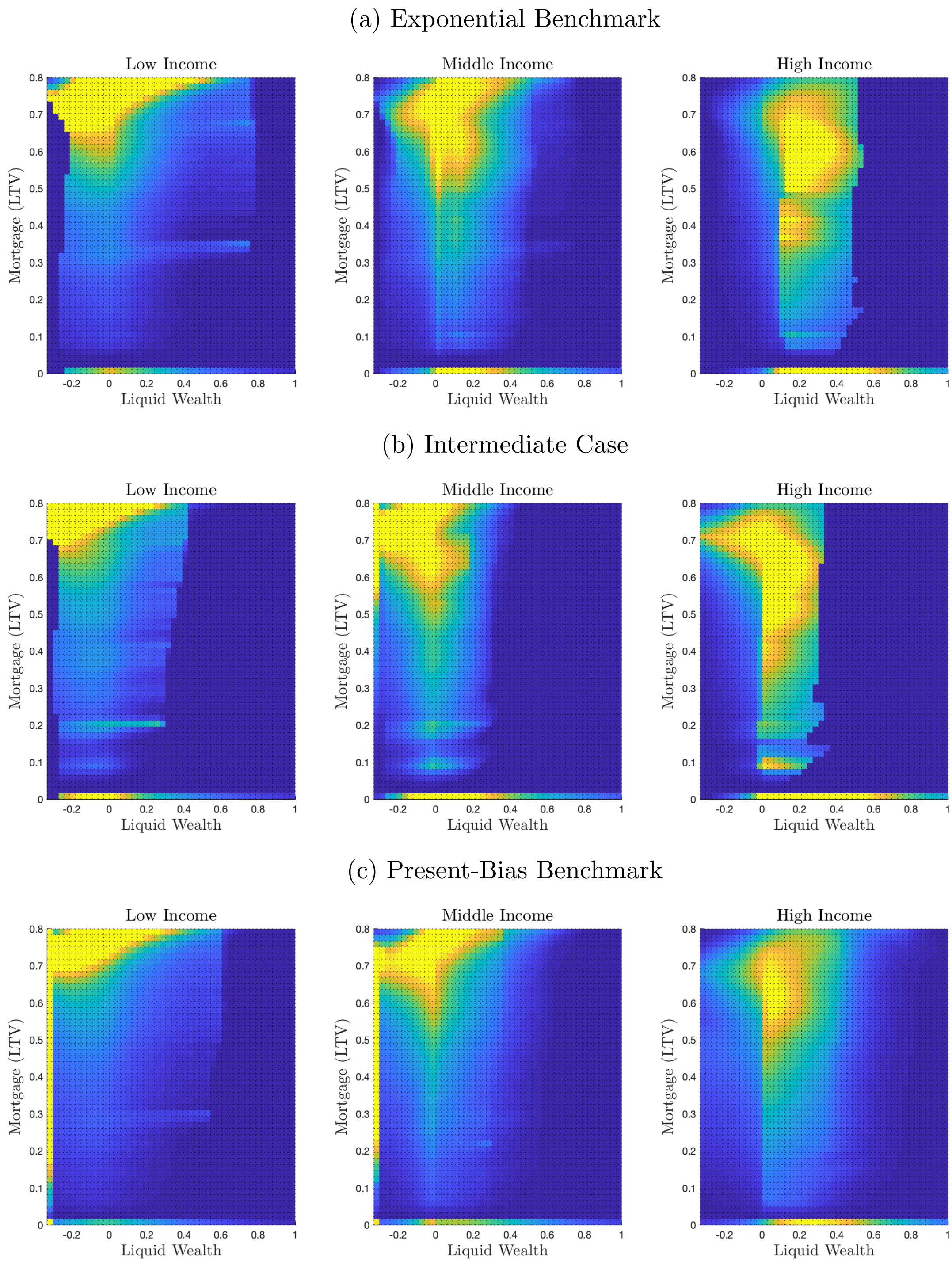


Figure 12: Steady State Distribution.

Notes: For the three calibration cases, this figure presents the full steady state distribution over income, liquid wealth, and mortgage debt. Dark blue regions are rarely encountered, while light yellow regions feature large masses of households.

E.3 Monetary Policy: Refinancing Dynamics

Figure 13 plots the adjustment regions following an interest rate cut from 1% to 0%. This figure replicates the phase diagrams in Figure 1, but now for the case of $r_t = 0\%$ and $r_t^m = 1\% + \omega^m$. Thus, Figure 13 plots the adjustment regions for households with a mortgage rate that is above the rate they can refinance into.

As in the main text, the red regions mark where households take a cash-out refinance and the blue regions mark where households prepay their mortgage. The gray regions indicate where households conduct a rate refinance, defined as the household increasing its mortgage balance by less than 5% during the refinance.

Relative to the steady state adjustment regions, the interest rate cut causes the red/gray refinancing regions to expand drastically. In particular, households with larger LTVs are more likely to refinance, since households with larger mortgages have more to gain by reducing their mortgage interest payments.

Table 6 presents details of the refinancing decision. The first row lists the share of households who find themselves in a refinancing region at the time of the interest rate cut. Conditional on refinancing, the second row lists the share of households who extract equity when refinancing. The next four rows list the share of households who have actually refinanced within 1 quarter, 1 year, 2 years, and 3 years following the interest rate cut. While refinancing is instant in the Exponential Benchmark and the Intermediate Case, procrastination means that refinancing occurs slowly in the Present-Bias Benchmark.

	Exponential	Intermediate	Present Bias
Share Refi Region (On Impact)	73.1%	68.5%	74.9%
(Share Cash Out)	81.0%	66.8%	77.3%
$\frac{1}{4}$ Year Realized Refi	75.2%	71.0%	13.6%
1 Year Realized Refi	80.0%	76.5%	42.0%
2 Year Realized Refi	84.5%	81.2%	62.7%
3 Year Realized Refi	87.8%	84.6%	74.3%
Average Refi Amount	0.31	0.17	0.29

Table 6: Refinancing Details.

Notes: For the three calibration cases, this table summarizes details of household refinancing decisions following an interest rate cut from 1% to 0%.

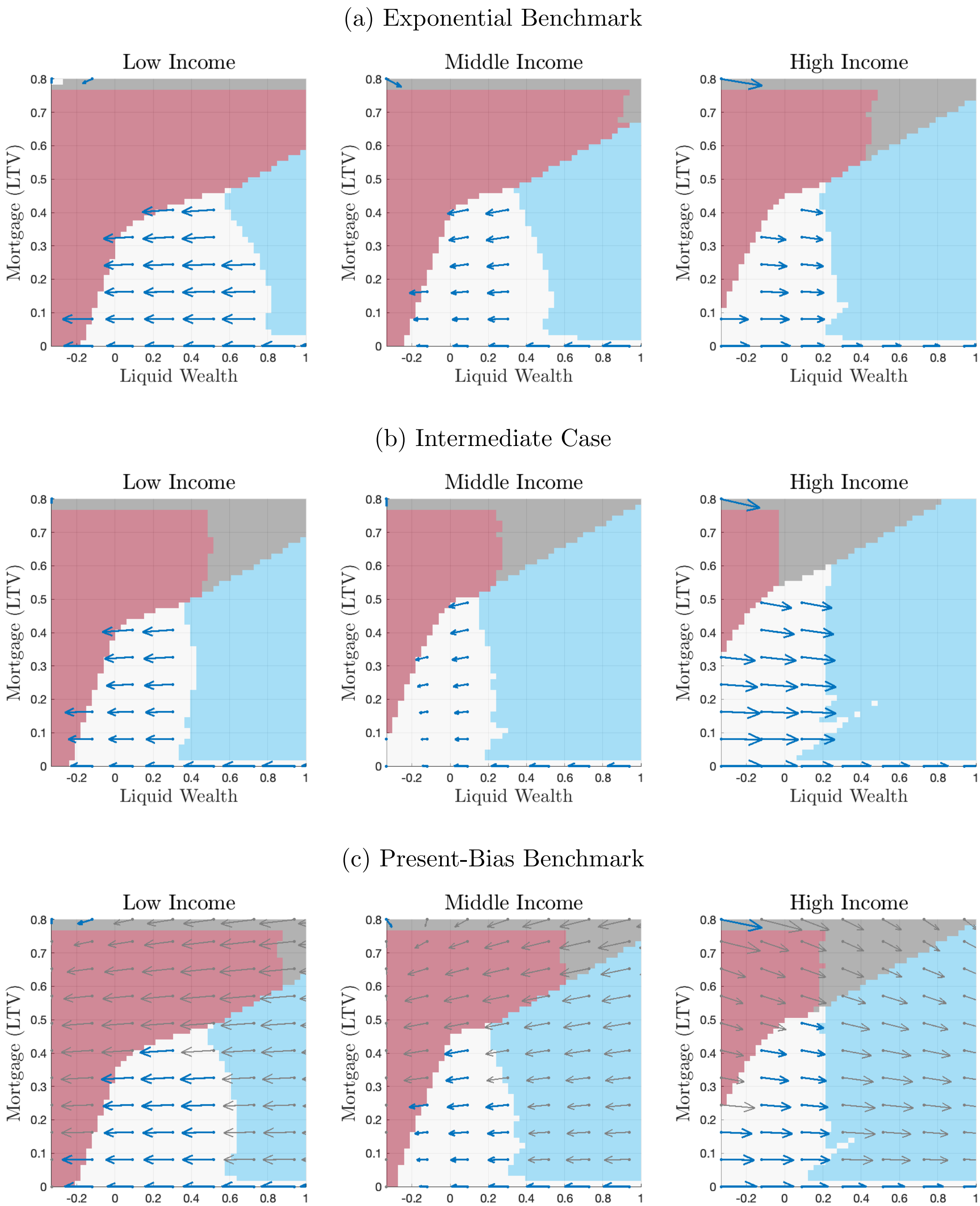


Figure 13: Rate-Cut Phase Diagrams.

Notes: For the three calibration cases, this figure presents the phase diagrams for households who can refinance into a lower mortgage rate following an interest rate cut from 1% to 0% (see Figure 1 for phase diagram details).

E.4 Monetary Policy and Refinancing Procrastination

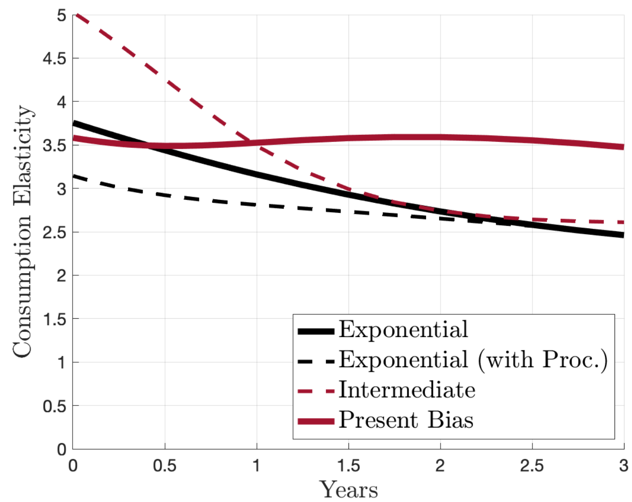


Figure 14: Consumption Response to Monetary Policy: Procrastination Sensitivity.
Notes: This figure adds a fourth case to the benchmark monetary policy analysis (Figure 6). This fourth case augments the Exponential Benchmark case with refinancing procrastination. The dashed black presents the consumption response to monetary policy in this exponential calibration with refinancing procrastination.

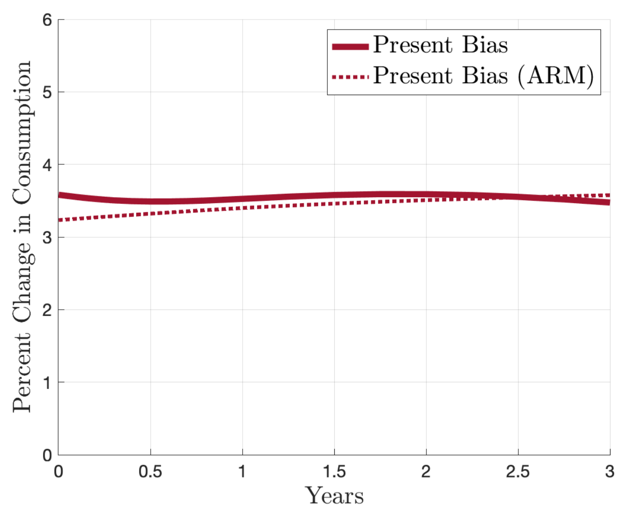


Figure 15: Monetary Policy Under FRMs Versus ARMs.
Notes: For the Present-Bias Benchmark calibration, this figure compares the consumption response to monetary policy under FRMs (solid line) versus ARMs (dotted line). The interest rate is cut by 2% in the ARM experiment, compared to 1% in the FRM experiment, since monetary policy produces larger movements in ARM rates than long-duration FRM rates.

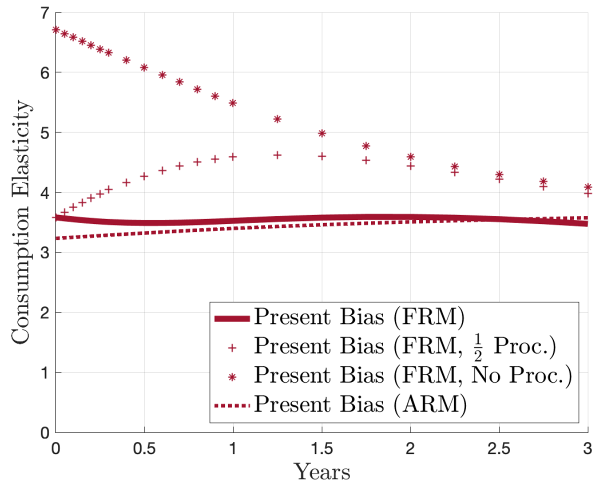


Figure 16: Monetary Policy with Procrastination Reduction.

Notes: This figure presents the consumption response to monetary policy in the Present-Bias Benchmark across varying levels of refinancing procrastination. The +'s assume that policymakers are able to halve the expected duration of procrastination at the time of the rate cut. The $*$'s make refinancing immediate at the time of the rate cut. The baseline consumption response under FRMs (solid red line) and ARMs (dotted red line) are presented for comparison.

E.5 Details on Aggregate House Price and Income Shocks

This section provides additional results for the analysis in Section 6.

House Price Shocks. Section 6.1 of the main text discusses the sensitivity of monetary policy to house price shocks. Here we provide further details on the fiscal policy experiment. Figure 17 plots the consumption response to fiscal stimulus in the negative (left) and positive (right) shock case. The corresponding MPCs are reported below in Table 7.

As the left panel of Figure 17 illustrates, the negative house price shock weakens the consumption response in the Present-Bias Benchmark over the first quarter, but strengthens the consumption response thereafter. The right panel of Figure 17 shows that the opposite is true of fiscal policy following a positive house price shock. In both cases, present bias strongly amplifies the consumption response to fiscal policy.

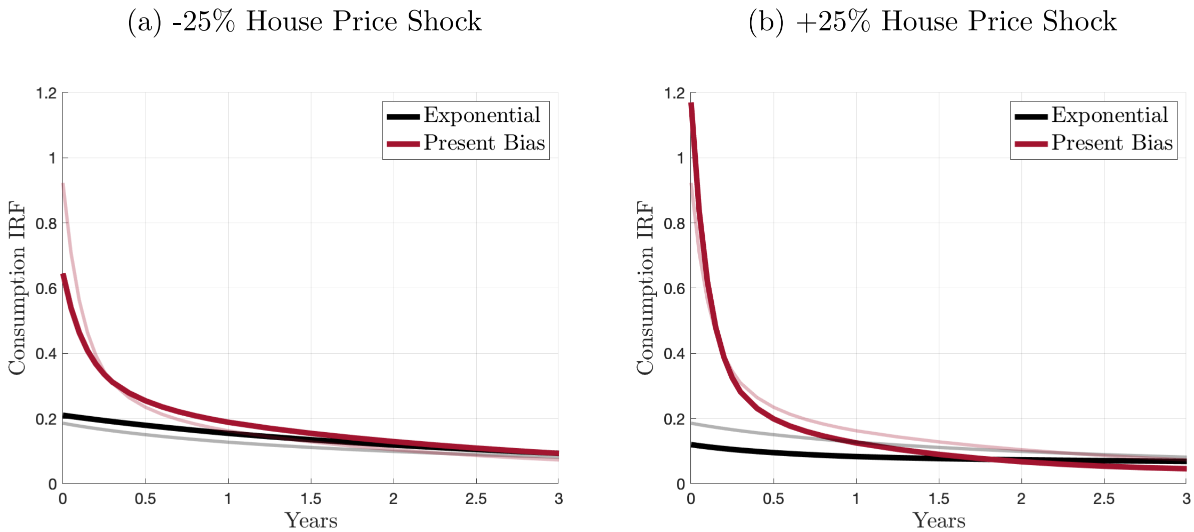


Figure 17: Fiscal Policy and House Price Shocks.
Notes: This figure plots the IRF of aggregate consumption to a \$1,000 fiscal transfer that immediately follows a house price shock of -25% (left) or +25% (right). The transparent lines plot to the baseline case in Figure 4, and are included for reference.

	Exponential	Present Bias
Baseline (No Shocks)		
1 Year MPC	15%	28%
2 Year MPC	26%	41%
3 Year MPC	35%	49%
-25% House Price Shock		
1 Year MPC	18%	28%
2 Year MPC	31%	43%
3 Year MPC	41%	53%
+25% House Price Shock		
1 Year MPC	10%	26%
2 Year MPC	17%	35%
3 Year MPC	24%	40%

Table 7: Fiscal Policy MPCs and House Price Shocks.
Notes: This table presents aggregate MPCs out of a \$1,000 fiscal transfer that is given immediately after a ± 25% house price shock.

Income Shocks. Section 6.2 of the main text outlines the effect of aggregate income shocks on monetary and fiscal policy. For this aggregate income shock experiment, the left panel of Figure 18 plots the consumption response to monetary policy and the right panel plots the consumption response to fiscal policy. As described in the main text, the consumption response is almost identical to the baseline case in Section 5.

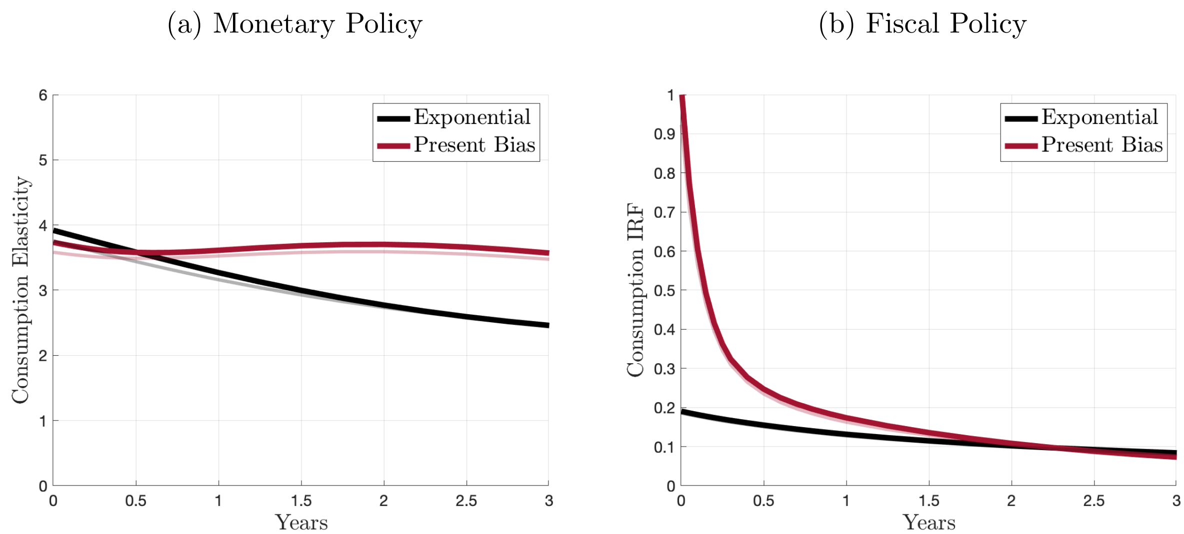


Figure 18: Fiscal and Monetary Policy Following a Negative Income Shock.
Notes: This figure plots the consumption response to monetary (left) and fiscal (right) policy that is implemented immediately following a transitory 5% decline in aggregate income.