

The Impact of Competition on Prices with Numerous Firms*

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Abstract

Many existing homogeneous-good markets exhibit large mark-ups, including some markets with many competing firms. We analyze simple monopolistic-competition models and show that idiosyncratic demand shocks driven by standard noise distributions can produce large equilibrium markups that are insensitive to the degree of competition. For example, with Gaussian noise and n firms, markups are proportional asymptotically to $1/\sqrt{\ln n}$; consequently, a hundred-fold increase in n , from 10 to 1000 competing firms, only halves the equilibrium markup. The elasticity of the markup with respect to n asymptotically equals the tail exponent from extreme value theory. Only noise distributions with extremely thin tails have negative asymptotic markup elasticities.

1 Introduction

The effect of competition on prices is one of the central questions in economics. Classical equilibrium models – for instance, Bertrand and Cournot competition – imply that competition

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quickly lowers prices. For example, when homogeneous consumers face a Cournot market, equilibrium markups are proportional to $1/n$, where n is the number of competing firms. Hence, doubling the number of firms halves the equilibrium mark-up.

In practice, however, markets with highly homogeneous goods and many competitors sometimes exhibit robustly high mark-ups. For example, Hortacsu and Syverson (2004) document high mark-ups in the mutual fund market, even in asset classes with hundreds of competing funds. Ausubel (1991) and Stango (2000) show that interest rates on credit cards have been much greater than the cost of funds, despite the presence of hundreds of competing card-issuing banks.

In this paper, we argue that high mark-ups are a robust feature of monopolistically competitive markets. We focus on random demand models, where consumer choice is influenced by random demand shocks, i.e., *noise* (Luce 1959, McFadden 1981, Anderson et al. 1992). We show that random demand models with standard (thin-tailed) ‘noise’ distributions predict high markups, even when goods are homogeneous *and* there is a large number of competing firms: increased competition only weakly drives down equilibrium mark-ups that arise from such noise.

Using extreme value theory (EVT), we develop tools that provide explicit expressions for equilibrium prices in symmetric random-utility models.¹ Explicit expressions for equilibrium markups have previously been derived only for some specific distributions of noise. In the previously studied special cases, equilibrium markups turn out to be either completely unresponsive or extremely responsive to competition. For instance, consider the Perloff-Salop (1985) model of competition, and assume that consumer noise has an exponential density or a logit (i.e., Gumbel) density. In this case, markups converge to a strictly positive value as the number of competing firms n goes to infinity: asymptotic markups have zero elasticity with respect to n (Perloff and Salop 1985, Anderson et al. 1992). In contrast, when noise is uniformly distributed, markups are proportional to $1/n$: markups have unit elasticity and thus decrease strongly with n (Perloff and Salop 1985).

These special cases — exponential, logit, and uniform — are appealing for their analytic tractability rather than their realism. In comparison to the Gaussian distribution, the exponential and logit cases have relatively fat tails while the uniform case has no tails. We

¹Empirical IO models no longer use symmetric, random-utility models (e.g., see the critiques in Bajari and Benkard 2003 and Armstrong 2015). Consequently, our techniques cannot be directly applied to the structural models that are most frequently used in empirical IO. This paper illustrates a mechanism that now needs to be extended to other settings.

seek to understand how prices respond to competition in the general case; in particular, for empirically realistic noise distributions.

The current paper solves the tractability problem for general noise distributions and derives a tractable formula for equilibrium markups. We show that markups are asymptotically proportional to $1/(nf[F^{-1}(1-1/n)])$, where F is the cumulative distribution function (CDF) of the noise and $f = F'$ is the corresponding distribution function. Moreover, we show that this markup turns out to be almost equivalent to the markup obtained under limit pricing. In other words, the markup is asymptotically proportional (and often equal) to the expected gap between the highest draw and second highest draw in a sample of n random draws of noise. Thus for large n , prices are pinned down by the tail properties of the distribution of taste shocks.

The Gaussian case – the leading thin-tailed case – is illustrative. In our setting of consumer choice, the Gaussian distribution is a natural benchmark: if a consumer receives many small, idiosyncratic influences on his preferences or beliefs, then (under appropriate assumptions) the sum of these influences produces a Gaussian-distributed ‘taste’ shock.² No closed-form solutions for equilibrium markups associated with Gaussian noise have previously been derived. We show that markups in the Gaussian case are asymptotically proportional to $1/\sqrt{\ln n}$. This implies that an increase in the number of competing firms from 10 to 1000 firms results in only a halving of the equilibrium markup. In contrast, with Cournot competition, such an increase would result in the markup becoming 100 times smaller. This example shows that competition with plausible noise distributions may only exert weak pressure on prices (even in the extreme case of homogeneous goods).

Further, we show that insensitive prices are the norm rather than the exception. Specifically, we find that the elasticity of the markup with respect to the number of firms asymptotically equals the EVT tail exponent of the noise distribution, a magnitude that is easy to calculate. Using this result, we show that markups have zero asymptotic elasticity for a wide range of noise distributions. Only noise distributions with extremely thin or no tails (like the uniform distribution), or very heavy tails (like the Pareto), have markups with asymptotic elasticities different from zero.

Moreover, our analysis implies that for distributions in the very heavy-tailed class (including subexponential distributions like the log-normal and power-law distributions like the Pareto distribution), mark-ups *increase* as the number of competing firms increase. This find-

²As implied by various versions of the central limit theorem; see, for example, Feller (1971, p. 262).

ing is closely connected to those of Weyl and Fabinger (2013) and Quint (2015), who show how comparative statics of pricing behavior hinge crucially on log-concavity of the demand function; relating this insight to our results, Weyl and Fabinger (2013) point out that competition increases (decreases) prices if the distribution of consumer valuations is log-convex (log-concave).³ These papers precisely demarcate the boundary between price-increasing and price-decreasing competition. Complementing these papers, we quantify the impact of competition on prices for general demand functions. In fact, we show that for a wide range of distributions above and below the aforementioned boundary, prices are relatively insensitive to competition.

Importantly, our findings exhibit “detail-independence”. They hold for all of the monopolistic competition models that we consider: Perloff-Salop (1995), Sattinger (1984), Hart (1985). The Perloff-Salop, Sattinger, and Hart models differ in a host of important ways.⁴ Yet, these three models produce (asymptotically) the same equilibrium markup up to a scaling constant, for a wide range of different noise distributions. Such detail-independence permits a more robust analysis than would be possible if results depended on the specific properties of the demand specification.

Our analysis is agnostic about the source of noise in consumer choice – whether the noise reflects heterogenous preferences with normative validity or consumer confusion about product quality – so our results are relevant to both the classical literature on imperfect competition and the emerging literature on behavioral industrial organization. That said, we find the behavioral interpretation that noise arises from consumer mistakes particularly intriguing. Consumer errors in product evaluation may arise from a variety of mechanisms; let us briefly outline two hypotheses. First, firms may engage in obfuscation to confuse naive (boundedly rational) consumers about product quality;⁵ this point is developed in a number of recent papers, including Spiegler (2006), Gabaix and Laibson (2006), Ellison and Ellison (2009), Armstrong and Vickers (2012) and Heidhues, Koszegi and Murooka (2014a, 2014b).⁶ Second,

³Relatedly, a number of other papers focus on the point that prices may rise with more intense competition: see, e.g., Chen and Riordan (2008), as well as Rosenthal (1980), Bénabou and Gertner (1993), Bulow and Klemperer (2002), Carlin (2009), and Zhelobodko et al (2012) for perverse competitive effects generated by different microfoundations.

⁴For instance, in the Perloff-Salop model, consumers need to buy one unit of the good. In the Sattinger model, they allocate a fixed dollar amount to the good. The Hart model does not impose either constraint.

⁵Our basic model exogenously specifies the degree of “obfuscation”. In the online appendix, we augment our model to consider deliberate shrouding / obfuscation by sellers, and show that our key insights are preserved in this richer setting.

⁶Relatedly, other papers (e.g. Bordalo, Gennaioli, Shleifer 2015) emphasize the impact of endogenous

consumers may be influenced by a multitude of idiosyncratic behavioral cues in their decision-making. For example, a consumer who is evaluating a mutual fund may rely on otherwise uninformative ‘tips’ from his friends and family.

In addition to an understanding of the economics of random demand models, the tools that we develop allow us to calculate the asymptotic behavior of integrals for a class of functions $h(x)$, of the form

$$\int h(x) f^k(x) F(x)^n dx, \quad k \geq 1. \quad (1)$$

This integral can be used to calculate the expected value of a function of the maximum of n random variables, or the gap between the maximum and the second largest value of those random variables. Using EVT, we derive robust approximations of this integral for large n .

These mathematical results have broad applications to various economic settings related to market and auction mechanisms in large economies.⁷ In particular, Mangin (2015a) analyzes an elegant model of frictional labour markets where firms compete via auction to hire workers, and points out that the present paper’s results may be applied to calculate the asymptotic value of key economic quantities such as the income share of labour. Using a closely related framework, Mangin (2015b) applies Theorem 3 of the present paper as a key step in deriving aggregate production functions from an underlying productivity distribution.⁸

The paper proceeds as follows. Section 2 presents the main economic result using the random-utility model of Perloff and Salop (1985), discusses welfare implications, and demonstrates the equivalence of our results to the limit-pricing and auction settings. Section 3 considers alternative models of monopolistic competition (Sattinger, 1984; and Hart, 1985), and shows that the details of the demand-side modeling matter little, or not at all, to markups. Section 4 presents the main mathematical result: an asymptotic approximation of a key integral that is needed to characterize economic environments in which extremes matter. We show that the tail of the noise distribution – captured by the tail exponent – is the crucial salience on market equilibrium.

⁷More generally, EVT techniques are important in many areas of economics, such as industrial organization and discrete choice (e.g., Luce 1959, McFadden 1981, Anderson et al. 1992, Dagsvik 1994, Bulow and Klemperer 2002, 2012, Weyl and Fabinger 2013, Dagsvik and Karlstrom 2005, Ibragimov and Walden 2010, and Armstrong 2014), international trade (e.g., Eaton and Kortum 2002, Bernard et al. 2003, and Chaney 2008, 2014), macroeconomics and growth (e.g., Gabaix 1999, 2011, Jones 2005, Luttmer 2007, and Acemoglu et al. 2012), systemic risk analysis (e.g., Jansen and De Vries 1991 and Ibragimov, Jaffee and Walden 2009, 2011) and auction theory (e.g., Hong and Shum 2004).

⁸In a separate application, Gabaix and Landier (2008) use some of our results to analyze the upper tail of the distribution of CEO talents.

determinant of prices. As many common noise distributions have a tail index of zero, our results imply that in a wide range of market contexts additional competition has little effect on prices, once the market goes beyond a small number of firms. Section 5 concludes.

We prove our main results (including Theorems 1 and 3) in Appendix A, and our other results in an online Appendix.

2 How Much Does Competition Affect Prices?

In this section, we introduce the model of oligopolistic competition from Perloff and Salop (1985). Postponing some of the mathematical elements of the proof (which are provided in Section 4), we report our key result: an asymptotic expression for price markups under oligopolistic competition. We then discuss implications and applications.

2.1 The Perloff-Salop Model

Our analysis is based on the Perloff-Salop (1985) model of monopolistic competition. There is a single representative consumer and an exogenously specified number of firms, n . The consumer must purchase exactly one unit of the good from one firm. He perceives that he will receive net utility $U_i = X_i - p_i$ by purchasing the good of firm i , where X_i is a noise term representing a random taste shock, i.i.d. across firms and consumers, and p_i is the price charged by firm i . Thus the consumer chooses to purchase the good that maximizes $X_i - p_i$. The timing is as follows:

1. Firms simultaneously set prices;
2. Random taste shocks are realized;
3. Consumers make purchase decisions;
4. Profits are realized.

The key economic object of interest is the price markup in a symmetric equilibrium, which we derive by solving the first-order condition for each firm's profit maximization problem. Firm i 's profit function is given by

$$\pi_i = (p_i - c)D(p_1, \dots, p_n; i) \tag{2}$$

where $D(p_1, \dots, p_n; i)$ is the demand function for firm i given the price vector (p_1, \dots, p_n) of the n goods, and where c is the marginal cost of production. The first order condition for profit maximization implies the following equilibrium markup in a symmetric equilibrium

$$p - c = -\frac{D(p, p; n)}{D_1(p, p; n)}. \quad (3)$$

Here p is the symmetric equilibrium price, $D(p, p'; n)$ denotes the demand function for a firm that sets price p when there are n goods and all other firms set price p' , and $D_1(p, p'; n) \equiv \partial D(p, p'; n) / \partial p$. Denote the markup $p - c$ in a symmetric equilibrium with n firms as μ_n .

In a symmetric-price equilibrium, the demand function of firm i is the probability that the consumer's surplus at firm i , $X_i - p_i$, exceeds the consumer's surplus at all other firms,

$$D(p_1, \dots, p_n; i) = \mathbb{P}(X_i - p_i \geq \max_{j \neq i} \{X_j - p_j\}) = \mathbb{P}(X_i \geq \max_{j \neq i} \{X_j\}). \quad (4)$$

Let M_n denote $\max\{X_1, \dots, X_n\}$, which has density $nf(x)F^{n-1}(x)$.⁹ Evaluation of (3) gives the following markup expression for the symmetric equilibrium of the Perloff-Salop model:

$$\mu_n = \frac{1}{n\mathbb{E}[f(M_{n-1})]} = \frac{1}{n(n-1) \int f^2(x)F^{n-2}(x) dx}. \quad (5)$$

Here F is the distribution function and f is the corresponding density of X_i .

Before proceeding to our analysis of the markup expression (5), let us briefly discuss our modeling approach. We use a stripped-down version of monopolistic competition for our analysis. In the model, the consumer's payoff function takes a simple additive form. We show in Section 3 that our results do not rely on this specification. There, we analyze two other models of monopolistic competition which feature (as in Perloff-Salop 1985) a representative consumer who has random i.i.d. taste shocks over producers, but differ in the form of consumer preferences. Our results from the present section are preserved in these alternative models, suggesting that the impact of competition on markups is independent of many of the institutional details of market competition.

A second feature of our model is that firms are completely symmetric ex ante, and thus each firm gets an equal $1/n$ market share in equilibrium. This assumption is strong, but enables tractable analysis; we conjecture that our main findings will be preserved when we

⁹Indeed, $\mathbb{P}(M_n \leq x) = \mathbb{P}(X_i \leq x \text{ for } i = 1 \dots n) = \mathbb{P}(X_i \leq x)^n = F(x)^n$.

extend the model to incorporate firm heterogeneity.

2.2 Extreme Value Theory: Some Basics

Now, we very briefly introduce some necessary machinery; we postpone some of the mathematical details to Section 4. As in Section 2.1, define $M_n \equiv \max_{i=1,\dots,n} X_i$ to be the maximum of n independent random variables X_i with distribution F . Also, define the counter-cumulative distribution function $\bar{F}(x) \equiv 1 - F(x)$.¹⁰ We are particularly interested in the connection between M_n and $\bar{F}^{-1}(1/n)$; informally (in analogy with the empirical distribution function), one may think of $\bar{F}^{-1}(1/n)$ as the “typical” value of M_n . In fact, the key to our analysis is to formalize this relationship between $\bar{F}^{-1}(1/n)$ and M_n for large n .

Our analysis is restricted to what we call *well-behaved* distributions:

Definition 1. *Let F be a distribution function with support on (w_l, w_u) , where $w_u \leq \infty$. We say F is well-behaved iff $f = F'$ is differentiable in a neighborhood of w_u , $\lim_{x \rightarrow w_u} \bar{F}/f = a$ exists with $a \in [0, \infty]$, and*

$$\gamma = \lim_{x \rightarrow w_u} \frac{d}{dx} \left(\frac{\bar{F}(x)}{f(x)} \right) \quad (6)$$

exists and is finite. We call γ the tail index of F .

Being well-behaved imposes a restriction on the right tail of F . The case $\gamma < 0$ consists of thin-tailed distributions with right-bounded support such as the uniform distribution. The case $\gamma = 0$ consists of distributions with tails of intermediate thickness. A wide range of economically interesting distributions fall within this domain, ranging from the relatively thin-tailed Gaussian distribution to the relatively thick-tailed lognormal distribution, as well as other distributions in between, such as the exponential distribution. The case $\gamma > 0$ consists of fat-tailed distributions such as the Pareto (power-law) and the Fréchet distributions.

Being well-behaved in the sense of Definition 1 is not a particularly strong restriction. It is satisfied by most distributions of interest, and is easy to verify.¹¹ In Section 2.3, Table 1 lists a number of popular densities and the corresponding tail index γ . Note that distributions with an exponential-like right tail all have $\gamma = 0$.

¹⁰Strictly speaking, we abuse notation in cases where F is not strictly increasing by using $\bar{F}^{-1}(t)$ to denote $\bar{F}^{\leftarrow}(t) = F^{\leftarrow}(1-t)$, where $F^{\leftarrow}(t) = \inf \{x \in (w_l, w_u) : F(x) \geq t\}$ is the generalized inverse of F (Embrechts et al. 1997, p.130). This is for expositional convenience: our results hold with the generalized inverse as well.

¹¹Condition (6) is well-known in the EVT literature as a second-order von Mises condition.

2.3 How do markups change with competition?

The next theorem is our key result: it characterizes, asymptotically, the equilibrium markup as a function of the noise distribution and the number of competing firms. For this result, we make a few mild assumptions. We assume that F is well-behaved, $f^2(x)$ is $[w_l, w_u]$ -integrable, and the tail index $-1.45 \leq \gamma \leq 0.64$.¹² Our main economic result is the following.¹³

Theorem 1. *The symmetric equilibrium markups in the Perloff-Salop model is, asymptotically (for $n \rightarrow \infty$),*

$$\mu_n \sim \frac{1}{nf \left(\bar{F}^{-1} \left(\frac{1}{n} \right) \right) \Gamma(\gamma + 2)}.$$

where $\bar{F}(x) \equiv 1 - F(x)$, and $\Gamma(t) \equiv \int_0^\infty y^{t-1} e^{-y} dy$ is the standard Gamma function.

Theorem 1 has a number of striking economic implications. Most directly, it allows us to characterize the equilibrium markup for various noise distributions. See Table 1.

The distributions in Table 1 are generally presented in increasing order of tail fatness. For the uniform distribution, which has the thinnest tails, the markup is proportional to $1/n$. This is the same equilibrium markup generated by the Cournot model. However the uniform and Cournot cases are unrepresentative of the general picture. For the distributions reported in Table 1, γ is bounded below by -1 , so the uniform distribution is a boundary case.

For the distributions with the fattest tails, markups paradoxically *rise* as the number of competitors *increases*.¹⁴ Intuitively, for fat-tailed noise, as n increases, the difference between the best draw and the second-best draw, which is proportional to $nf \left(\bar{F}^{-1} (1/n) \right)$, increases with n (see Section 2.5 below).

The discussion so far suggests that the markup function depends on the nature of the distribution f . In fact, we can state this point more precisely. The following proposition shows that the tail parameter γ in (6) has a concrete economic implication: it is the asymptotic elasticity of the markup with respect to the number of firms. In other words, interpreting n as

¹²This is the range over which the second order condition holds (see the online appendix for details); the first order condition holds whenever $\gamma > -2$. Note that this assumption on γ is not restrictive: it permits thin-tailed distributions such as the Weibull, and all (fat-tailed) Pareto distributions with finite variance.

¹³The proof relies on Theorem 3, proven later; for expositional convenience, we start with the main economic results.

¹⁴However, even though markups rise with n , profits per firm go to zero (keeping market size constant) since firm prices scale with n^γ but sales volume per firm is proportional to $1/n$ in the Perloff-Salop case and $1/n^{1+\gamma}$ in the Sattinger case.

Table 1: **Asymptotic Expressions for Markups**

This table lists asymptotic markups (under symmetric equilibrium) for the Perloff-Salop model for various noise distributions as a function of the number of firms n . f specifies the density function, and γ specifies the distribution's tail index. Distributions are listed in order of increasing tail fatness. Asymptotic approximations are calculated using Theorem 1 except where the markup can be exactly evaluated.

Name of Distribution	f	γ	μ_n	$\lim_{n \rightarrow \infty} \mu_n$
Uniform	$1, x \in [-1, 0]$	-1	$1/n$	0
Bounded Power Law	$\alpha (-x)^{\alpha-1}$ $\alpha > 0, x \in [-1, 0]$	$-1/\alpha$	$\frac{\Gamma(1-1/\alpha+n)}{\alpha\Gamma(2-1/\alpha)\Gamma(1+n)} \sim \frac{n^{-1/\alpha}}{\alpha\Gamma(2-1/\alpha)}$	0
Weibull	$\alpha (-x)^{\alpha-1} e^{-(-x)^\alpha}$ $\alpha \geq 1, x < 0$	$-1/\alpha$	$\frac{1}{\alpha\Gamma(2-1/\alpha)} \frac{n^{1-1/\alpha}}{n-1} \sim \frac{n^{-1/\alpha}}{\alpha\Gamma(2-1/\alpha)}$	0
Gaussian	$(2\pi)^{-1/2} e^{-x^2/2}$	0	$\sim (2 \ln n)^{-1/2}$	0
Rootzen class, $\phi > 1$	$\kappa\lambda\phi x^{\alpha+\phi-1} e^{-x^\phi}$	0	$\sim \frac{1}{\phi\lambda^{1/\phi}} (\ln n)^{1/\phi-1}$	0
Gumbel	$\exp(-e^{-x} - x)$	0	$\frac{n}{n-1}$	1
Exponential	$e^{-x}, x > 0$	0	1	1
Rootzen Gamma	$\tau x^{\tau-1} e^{-x^\tau}$ $x > 0, \tau < 1$	0	$\sim \frac{1}{\tau} (\ln n)^{1/\tau-1}$	∞
Log-normal	$\frac{\exp(-2^{-1} \log^2 x)}{x\sqrt{2\pi}}$ $x > 0$	0	$\sim \frac{1}{\sqrt{2 \ln n}} e^{\sqrt{2 \ln n}}$	∞
Power law	$\alpha x^{-\alpha-1}$ $\alpha > 1, x \geq 1$	$1/\alpha$	$\frac{\Gamma(1+1/\alpha+n)}{\alpha\Gamma(2+1/\alpha)\Gamma(1+n)} \sim \frac{n^{1/\alpha}}{\alpha\Gamma(2+1/\alpha)}$	∞
Fréchet	$\alpha x^{-\alpha-1} e^{-x^{-\alpha}}$ $\alpha > 1, x \geq 0$	$1/\alpha$	$\frac{1}{\alpha\Gamma(2+1/\alpha)} \frac{n^{1+1/\alpha}}{n-1} \sim \frac{n^{1/\alpha}}{\alpha\Gamma(2+1/\alpha)}$	∞

a continuous variable, the markup behaves locally as $\mu \sim kn^\gamma$. We assume that the conditions in Theorem 1 hold, and further that $\log F(x) f^2(x)$ is $[w_l, w_u)$ -integrable.

Proposition 1. *The asymptotic elasticity of the Perloff-Salop markup with respect to the number of firms n is*

$$\lim_{n \rightarrow \infty} \frac{n}{\mu_n} \frac{d\mu_n}{dn} = \gamma.$$

For taste shocks with distributions fatter than the uniform ($\gamma > -1$), Proposition 1 shows that the mark-up falls more slowly than $1/n$. In particular, $\gamma = 0$ corresponds to the case of intermediate tail thickness; it includes distributions ranging from the Gaussian to the lognor-

mal (again, see Table 1). This range encompasses the cases of price-decreasing competition (e.g., Gaussian) as well as price-increasing competition (e.g., log-normal). Within this range, Proposition 1 tells us that markup elasticity is asymptotically zero. In other words, competition has remarkably little effect on markups for such distributions.

Markup sensitivity: an example Extremely thin-tailed distributions (e.g. uniform) and extremely fat-tailed distributions (e.g. power-laws) are atypical cases in Table 1. Most of the distributional cases imply that competition typically has *little* impact on markups. For instance with Gaussian noise, the markup μ_n is proportional to $1/\sqrt{\ln n}$. Accordingly, μ_n converges to zero, but this convergence proceeds at a glacial pace. Indeed, the elasticity of the markup with respect to n converges to zero.

To illustrate this slow convergence, we calculate μ_n when noise is Gaussian for a series of values of n . Table 2 shows that in the models we study and with Gaussian noise, a highly competitive industry with $n = 1,000,000$ firms will retain a third of the markup of a highly concentrated industry with only $n = 10$ competitors. We also compare markups in our monopolistic competition models to those in the Cournot model, which features markups proportional to $1/n$ and a markup elasticity w.r.t. n of -1 (note that this is equal to markups in the Perloff-Salop model with uniformly distributed noise.)

Table 2: **Markups with Gaussian Noise and Uniform Noise**

Markups are calculated for (i) the symmetric equilibrium of the Perloff-Salop model for Gaussian noise and (ii) under Cournot Competition, for various values of the number of firms n . n is the number of firms in the market. Markups are normalized to equal one when $n = 10$.

n	Markup with Gaussian noise	Markup under Cournot Competition
10	1	1
100	0.61	0.1
1,000	0.47	0.01
10,000	0.40	0.001
100,000	0.35	0.0001
1,000,000	0.32	0.00001

More generally, in cases with moderate fatness of the tails, such as the Gumbel (i.e.,

logit), exponential, and log-normal densities, the markup again shows little (zero asymptotic) response to changes in n . Nevertheless, the markups become unbounded for the log-normal distribution. Finally, the case of Bounded Power Law noise shows that an infinite support is not necessary for our results. In this case the markup is proportional to $n^{-1/\alpha}$ and markup decay remains slow for large α .

In practical terms, these results imply that in markets with noisy demand we should not assume that increased competition will dramatically reduce markups. The mutual fund industry exemplifies such stickiness. Currently, 10,000 mutual funds are available in the U.S. and many of these funds offer nearly identical portfolios. Even in a narrow class of homogenous products, such as medium capitalization value stocks or high-yield bonds, it is normal to find 100 or more competing funds. Despite the large and rising number of competitors in such sub-markets, Hortacsu and Syverson (2004) report that mutual funds still charge high annual fees: more than 1% of assets under management for most industry sectors. These fees have changed very little even as the number of homogeneous competing funds has increased by a factor of 10 over the past several decades. We note that the issue is more general: for instance, Henderson and Pearson (2011) find that structured equity products also have robustly high mark-ups, and hypothesize that this is related to investor confusion about product quality. Bergresser, Chalmers and Tufano (2009) find that mutual funds sold by brokers have anomalously high fees and low net-of-fee returns. Another complementary explanation is that investors like the psychological comfort given by specific mutual fund brokers (Gennaioli, Shleifer, and Vishny 2015); this may be a potential mechanism for microfounding the random demand in our model.

2.4 Consumer Surplus

The random demand framework is sometimes criticized for generating an unrealistically high value for consumer surplus and social surplus. Indeed, if the noise distribution is unbounded (and the noise is treated as normatively meaningful taste shocks), then total consumer surplus tends to ∞ as the number of firms increases. Our analytical results allow us to examine this criticism.

To perform welfare analysis, in this subsection we interpret taste shocks as capturing normatively meaningful preference heterogeneity among consumers; so our measure of consumer surplus is simply $X_i - p_i$ where X_i is the consumer taste shock and p_i is the price for the purchased good. In this setting, expected consumer surplus is $\mathbb{E}[M_n] - p$ and expected social

surplus is $\mathbb{E}[M_n] - c$, where p is the equilibrium price and $M_n \equiv \max_{i=1,\dots,n} X_i$ is the largest quality draw from n firms. For brevity, we restrict ourselves to the case with unbounded distributions and $\gamma \geq 0$.

We can show that $\mathbb{E}[M_n] \sim \Gamma(1 - \gamma) \bar{F}^{-1}(1/n)$ for $\gamma \geq 0$.¹⁵ For all the distributions that we study except the unbounded power law case, $\bar{F}^{-1}(1/n)$ rises only slowly with n . Hence, even for unbounded distributions, and large numbers of producers, surplus can be quite small. For example, for the case of Gaussian noise when consumer preferences have a standard deviation of \$1, $\bar{F}^{-1}(1/n) \sim \sqrt{2 \ln n}$; with a million toothpaste producers, consumer surplus averages no more than \$5.25 per tube. Hence, in many instances, the framework — even with unbounded distributions — does not generate counterfactual predictions about surplus or counterfactual predictions about the prices that cartels would set.

2.5 Limit Pricing: An Alternative Interpretation

We now discuss an alternative model of oligopolistic competition, sometimes called “limit pricing”, which has proven to be very useful in trade and macroeconomics (e.g., Bernard et al., 2003; see also Auer and Chaney, 2009). The price-setting mechanism in the limit pricing model is remarkably simple, yet it produces (asymptotically) the same markups as the Perloff-Salop (1985) model. This equivalence result implies that a similar logic underlies the equilibrium markups for these models, and thus generates a simple but useful interpretation of our economic results.

In the limit pricing model, each firm i draws i.i.d. quality shock X_i . Firms simultaneously sets prices p_i *after* observing other firms’ quality shocks. (This is in contrast with the Perloff-Salop 1985 model, where prices are set *before* taste shocks are observed.) The representative consumer purchases one unit of the good, and picks the firm which maximizes $X_i - p_i$. As before, call $M_n = \max_{i=1\dots n} X_i$ the largest quality draw from the n firms, and S_n the second-largest draw. In the competitive equilibrium, the firm with the highest quality, M_n , gets all the market share, and sets a markup of $\mu_n^{LP} = M_n - S_n$. This is just enough to take all the market away from the firm with the second-highest quality.

The next Proposition analyzes the equilibrium markup under Limit Pricing. We assume that F is well-behaved with tail index $\gamma < 1$, and that $xf(x)$ is $[w_l, w_u]$ -integrable.

¹⁵This result is an immediate application of Theorem 3, which we present in Section 4.

Proposition 2. Call M_n and S_n , respectively, the largest and second largest realizations of n i.i.d. random variables with CDF F . Then limit pricing markup is $\mu_n^{LP} = M_n - S_n$, and

$$\mathbb{E} [\mu_n^{LP}] \sim_{n \rightarrow \infty} \frac{\Gamma(1 - \gamma)}{nf \left(\bar{F}^{-1} \left(\frac{1}{n} \right) \right)}. \quad (7)$$

Notice that this markup is asymptotic to the markup from Theorem 1. This suggests the following intuition for Theorem 1: to set its optimum price, a firm conditions on getting the largest draw, then evaluates the likely draw of the second highest firm and engages in limit pricing, where it charges a markup equal to the difference between its draw and the next highest draw: $\mathbb{E} [\mu_n^{LP}] \approx M_n - S_n$. (This is analogous to the analysis of a first price sealed bid auction.) In fact, this reasoning gets us approximately the correct answer: observe that $\mathbb{E} [\bar{F}(M_n)] \simeq \frac{1}{n+1}$ and $\mathbb{E} [\bar{F}(S_n)] \simeq \frac{2}{n+1}$, which suggests that M_n (the highest draw) will be close to $\bar{F}^{-1} \left(\frac{1}{n} \right)$ and that S_n (the second-highest draw) will be close to $\bar{F}^{-1} \left(\frac{2}{n} \right)$. So,

$$\begin{aligned} \mathbb{E} [\mu_n^{LP}] &\approx M_n - S_n \approx \bar{F}^{-1} (1/n) - \bar{F}^{-1} (1/n + 1/n) \\ &\approx -\frac{1}{n} \cdot \left(\bar{F}^{-1} \right)' (1/n) \quad \text{by Taylor expansion} \\ &= \frac{1}{nf \left(\bar{F}^{-1} (1/n) \right)}. \end{aligned}$$

In fact, revisiting Theorem 1, we see that this heuristic argument generates the right approximation for the Perloff-Salop markups when $\gamma = 0$ (e.g. Gaussian, logit (Gumbel), exponential, and lognormal distributions), and that the approximation remains accurate up to a corrective constant for the other distributions.

2.5.1 An Application to Auctions

Our mathematical results can also be applied to the analysis of auctions. Consider a second-price auction with a single good and n bidders where each bidder i privately values the good at X_i , which is i.i.d. with CDF F . It is well-known that if F is strictly increasing on (w_l, w_u) , then in equilibrium each bidder bids his private valuation; the bidder with the highest valuation M_n wins and pays the second-highest valuation S_n . Proposition 2 then immediately implies

that the expected surplus for the winner of the auction is¹⁶

$$\mathbb{E}[M_n - S_n] \sim_{n \rightarrow \infty} \frac{\Gamma(1 - \gamma)}{nf(\bar{F}^{-1}(1/n))}. \quad (8)$$

Other key quantities are also easily derived: for example, the seller's expected revenue is

$$\mathbb{E}[S_n] \sim_{n \rightarrow \infty} \bar{F}^{-1}(1/n) \Gamma(2 - \gamma) \text{ if } w_u = \infty. \quad (9)$$

For some applications of these results to large auction settings, see, e.g., Mangin (2015a).

3 Detail-Independence

This section demonstrates the robustness of our main findings from Section 2 to alternative assumptions about consumer preferences.

3.1 Alternative Models

We briefly describe two alternative random demand models (mentioned in Section 2.1), then show that these models also obey the asymptotic markup rule of Theorem 1. These models differ from Perloff-Salop (1985) in the specification of consumer preferences, but otherwise share common features: there is a single representative consumer and n firms, indexed as $i = \{1, \dots, n\}$, in a monopolistically competitive market. The timing is also the same: firms set prices simultaneously, before they learn their own (or others') quality shocks. As before, p_i is the price of good i , and the random shocks X_i associated with each good i are i.i.d. randomly distributed with distribution function F .

Sattinger (1984) analyzes the case of multiplicative random demand, where consumers demand a fixed dollar amount. There are two types of goods. Besides the monopolistically competitive market, there is separately a composite good purchased from an industry with homogenous output. Our focus is on markups in the monopolistically competitive market.

¹⁶The case $\gamma \neq 0$ of result (8) appeared in Caserta (2002, Prop. 4.1), the proof of which relied on a different argument.

The consumer has utility function

$$U = Z^{1-\theta} \left[\sum_{i=1}^n A_i Q_i \right]^\theta, \quad (10)$$

where Z is the quantity of the composite good, $A_i = \exp(X_i)$ is the random taste shock, and Q_i is the quantity consumed of good i . The consumer faces the budget constraint $y = qZ + \sum_i p_i Q_i$, where y is the consumer's endowment and q is the price of the composite good. In the online appendix, we show that the equilibrium markup in this model is

$$\frac{\mu_n^{Satt}}{c} = \frac{1}{n(n-1) \int f^2(x) F^{n-2}(x) dx} = \frac{1}{n \mathbb{E}[f(M_{n-1})]}. \quad (11)$$

Hart (1985) analyzes a richer setup where consumers' demand is flexible in quantity and value. In comparison, in the Perloff-Salop model, the quantity demanded is fixed; whereas in the Sattinger model, dollar expenditure is fixed. The consumer's utility function is:

$$U^{Hart} = \frac{\psi + 1}{\psi} \left(\sum_{i=1}^n A_i Q_i \right)^{\psi/(\psi+1)} - \sum_{i=1}^n p_i Q_i, \quad (12)$$

where $A_i = e^{X_i}$ is the associated random taste shock for good i and Q_i is the consumed quantity. The equilibrium markup of the Hart (1985) model is¹⁷

$$\frac{\mu_n^{Hart}}{c} = \frac{1}{\psi + (n-1) \frac{\int e^{\psi x} f^2(x) F^{n-2}(x) dx}{\int e^{\psi x} f(x) F^{n-1}(x) dx}} = \frac{1}{\psi + (n-1) \frac{\mathbb{E}[e^{\psi M_{n-1}} f(M_{n-1})]}{\mathbb{E}[e^{\psi M_{n-1}}]}}. \quad (13)$$

3.2 Comparing Equilibrium Markups

We now characterize equilibrium markups for the Sattinger (1984) and Hart (1985) models. As in Theorem 1, we assume that F is well-behaved, and that $f^2(x)$ is $[w_l, w_u]$ -integrable. For the Sattinger model, assume that $-1.45 \leq \gamma \leq 0.64$.¹⁸ For the Hart model with parameter ψ , assume that $-1 < \gamma \leq 0$; if $\gamma = 0$, we further require that $1 - \psi a > 0$.¹⁹ Denote

¹⁷Note that in the special case $\psi = 0$, by comparing (11) with (13), we see that $\mu_n^{Hart} = \mu_n^{Satt}$; that is, the Hart model generates the same demand functions and markups as the Sattinger model.

¹⁸As with the Perloff-Salop (1985) model, this is the range over which the second order condition holds (see the online appendix for details).

¹⁹For distributions violating this condition, no symmetric price equilibrium can be calculated in the Hart model because each firm would face infinite demand.

Table 3: **Asymptotic Expressions for Sattinger and Hart Markups**

This table reproduces Table 1 and adds asymptotic markups for the Sattinger and Hart models. μ_n , μ_n^{Satt} and μ_n^{Hart} are respectively the asymptotic markup expressions for the Perloff-Salop, Sattinger, and Hart models. Asymptotic approximations are calculated using Theorems 1 and 2 except where the markup can be exactly evaluated. The Hart markup is undefined for distributions fatter than the exponential.

Distribution	f	$\mu_n = \mu_n^{Satt}/c$	μ_n^{Hart}/c	$\lim_{n \rightarrow \infty} \mu_n$
Uniform	$1, x \in [-1, 0]$	$1/n$	$\sim 1/n$	0
Bounded Power Law	$\alpha(-x)^{\alpha-1}$ $\alpha > 0, x \in [-1, 0]$	$\frac{\Gamma(1-1/\alpha+n)}{\alpha\Gamma(2-1/\alpha)\Gamma(1+n)} \sim \frac{n^{-1/\alpha}}{\alpha\Gamma(2-1/\alpha)}$	$\sim \frac{n^{-1/\alpha}}{\alpha\Gamma(2-1/\alpha)}$	0
Weibull	$\alpha(-x)^{\alpha-1} e^{-(-x)^\alpha}$ $\alpha \geq 1, x < 0$	$\frac{1}{\alpha\Gamma(2-1/\alpha)} \frac{n^{1-1/\alpha}}{n-1} \sim \frac{n^{-1/\alpha}}{\alpha\Gamma(2-1/\alpha)}$	$\sim \frac{n^{-1/\alpha}}{\alpha\Gamma(2-1/\alpha)}$	0
Gaussian	$(2\pi)^{-1/2} e^{-x^2/2}$	$\sim (2 \ln n)^{-1/2}$		0
Rootzen class, $\phi > 1$	$\kappa\lambda\phi x^{a+\phi-1} e^{-x^\phi}$	$\sim \frac{1}{\phi\lambda^{1/\phi}} (\ln n)^{1/\phi-1}$		0
Gumbel	$\exp(-e^{-x} - x)$	$\frac{n}{n-1}$	~ 1	1
Exponential	$e^{-x}, x > 0$	1		1
Rootzen Gamma	$\tau x^{\tau-1} e^{-x^\tau}$ $x > 0, \tau < 1$	$\sim \frac{1}{\tau} (\ln n)^{1/\tau-1}$	—	∞
Log-normal	$\frac{\exp(-2^{-1} \log^2 x)}{x\sqrt{2\pi}}$ $x > 0$	$\sim \frac{1}{\sqrt{2 \ln n}} e^{\sqrt{2 \ln n}}$	—	∞
Power law	$\alpha x^{-\alpha-1}$ $\alpha > 1, x \geq 1$	$\frac{\Gamma(1+1/\alpha+n)}{\alpha\Gamma(2+1/\alpha)\Gamma(1+n)} \sim \frac{n^{1/\alpha}}{\alpha\Gamma(2+1/\alpha)}$	—	∞
Fréchet	$\alpha x^{-\alpha-1} e^{-x^{-\alpha}}$ $\alpha > 1, x \geq 0$	$\frac{1}{\alpha\Gamma(2+1/\alpha)} \frac{n^{1+1/\alpha}}{n-1} \sim \frac{n^{1/\alpha}}{\alpha\Gamma(2+1/\alpha)}$	—	∞

the Perloff-Salop, Sattinger and Hart markups as, respectively, μ_n , μ_n^{Satt} , and μ_n^{Hart} . The following theorem states that (up to a factor c) that all three markups are asymptotically equal; in fact, the Sattinger markup is exactly equal to the Perloff-Salop markup.

Theorem 2. *The symmetric equilibrium markups in the Perloff-Salop, Sattinger and Hart models are asymptotically*

$$\mu_n = \mu_n^{Satt}/c \sim \mu_n^{Hart}/c \sim \frac{1}{nf \left(\bar{F}^{-1} \left(\frac{1}{n} \right) \right) \Gamma(\gamma + 2)}. \quad (14)$$

with $\bar{F}(x) \equiv 1 - F(x)$.

Theorem 2 delivers the perhaps unexpected result that the Perloff-Salop (1985), Sattinger (1984), and Hart (1985) models generate asymptotically equal (up to a multiplicative constant) markups; see Table 3. Hence, detail-independence holds: equilibrium markups do not depend on the details of the model of competition. The key ingredient in the modeling is the specification of the noise distribution, rather than the details of the particular oligopoly model. In particular, these results suggest that the limit-pricing logic of Section 2.5 has broad applicability to random-demand models of monopolistic competition.

4 Methodological Results

This section presents our main mathematical results. Solving for the symmetric equilibrium outcome for distribution function F requires the evaluation of integrals of the form

$$\int x^j e^{\psi x} f^k(x) F(x)^{n-l} dx \quad (15)$$

where $k, l \geq 1$ and $j, \psi \geq 0$. For large n , such integrals mainly depend on the tail of the distribution F , which suggests that techniques from Extreme Value Theory (EVT) may be applied. (See Resnick (1987), and Embrechts *et al.* (1997) for an introduction to EVT.) We introduce concepts and notation in Section 4.1, then state our main results and briefly discuss the application to our markup results in Section 4.2. Proofs are in the Appendix.

4.1 Concepts and Notation from EVT

We start by developing the mathematical tools for asymptotically evaluating (15). Definition 1 ensures that the right tail of F behaves appropriately. To ensure that the integral (15) does not diverge, we also impose some restrictions on the rest of F .

Definition 2. Let $j : \mathbb{R} \rightarrow \mathbb{R}$ have support on (w_l, w_u) . The function $j(x)$ is $[w_l, w_u)$ -integrable iff

$$\int_{w_l}^w |j(x)| dx < \infty$$

for all $w \in (w_l, w_u)$.

For example, in Theorem 1 we require that f^2 be $[w_l, w_u)$ -integrable. Verification of this condition is typically straightforward; for example, $f^2(x)$ is $[w_l, w_u)$ -integrable if $f = F'$ is uniformly bounded.

Definition 3. A function $h : \mathbb{R}^+ \rightarrow \mathbb{R}$ is regularly varying at ∞ with index ρ if h is strictly positive in a neighborhood of ∞ , and

$$\forall \lambda > 0, \lim_{x \rightarrow \infty} \frac{h(\lambda x)}{h(x)} = \lambda^\rho. \quad (16)$$

We indicate this by writing $h \in RV_\rho^\infty$.

Analogously, we say that $h : \mathbb{R}^+ \rightarrow \mathbb{R}$ is regularly varying at zero with index ρ if, $\forall \lambda > 0, \lim_{x \rightarrow 0} h(\lambda x)/h(x) = \lambda^\rho$, and denote this by $h \in RV_\rho^0$. Intuitively, a regularly varying function $h(x)$ with index ρ behaves like x^ρ as x goes to the appropriate limit, perhaps up to logarithmic corrections. For instance, x^ρ and $x^\rho |\ln x|$ are regularly varying (with index ρ) at both 0 and ∞ . Much of our analysis will require the concept of regular variation; specifically, we will require that certain transformations of the noise distribution F be regularly varying. In the case $\rho = 0$, we say that h is *slowly varying* (for example, $\ln x$ varies slowly at infinity and zero).

Finally, following the notation of Definition 1, define

$$w_l = \inf\{x : F(x) > 0\} \quad \text{and} \quad w_u = \sup\{x : F(x) < 1\} \quad (17)$$

to be the lower and upper bounds of the support of F , respectively.

4.2 Core Mathematical Result, and Applications

Our core mathematical result documents an asymptotic relationship between M_n and $\bar{F}^{-1}(1/n)$.

Theorem 3. Let F be a differentiable CDF with support on (w_l, w_u) that is strictly increasing in a left neighborhood of w_u . Let $G : (w_l, w_u) \rightarrow \mathbb{R}$ be strictly positive in some left neighborhood of w_u . Suppose that $\hat{G}(t) \equiv G(\bar{F}^{-1}(t)) \in RV_\rho^0$ with $\rho > -1$, and that $|\hat{G}(t)|$ is integrable on $t \in (\bar{t}, 1)$ for all $\bar{t} \in (0, 1)$ (or, equivalently, $G(x)f(x)$ is $[w_l, w_u)$ -integrable in the sense of Definition 2). Then, for $n \rightarrow \infty$,

$$\mathbb{E}[G(M_n)] = \int_{w_l}^{w_u} nG(x)f(x)F(x)^{n-1}dx \sim G\left(\bar{F}^{-1}\left(\frac{1}{n}\right)\right)\Gamma(\rho+1) \quad (18)$$

where M_n is the largest realization of n i.i.d. random variables with CDF F .

The intuition for equation (18) follows. By definition of M_n , if X is distributed as F and if M_n and X are independent, then $\mathbb{P}[X > M_n] = 1/(n+1)$; that is, $\mathbb{E}[\bar{F}(M_n)] = 1/(n+1) \approx 1/n$. Consequently, we might conjecture (via heroic commutation of the expectations operator) that

$$\mathbb{E}[M_n] \approx \bar{F}^{-1}\left(\frac{1}{n}\right) \quad (19)$$

and more generally that $\mathbb{E}[G(M_n)] \approx G(\mathbb{E}[M_n]) \approx G(\bar{F}^{-1}(1/n))$. It turns out that this heuristic argument gives us the correct approximation, up to a correction factor $\Gamma(\rho+1)$.²⁰

We next present an intermediate result that is technically undemanding but will allow us to apply Theorem 3 to expressions of the form (15).

Lemma 1. *Let F be well-behaved with tail index γ . Then*

1. $f(\bar{F}^{-1}(t)) \in RV_{\gamma+1}^0$.
2. If $w_u = \infty$, then $\bar{F}^{-1}(t) \in RV_{-\gamma}^0$. If $w_u < \infty$, then $w_u - \bar{F}^{-1}(t) \in RV_{-\gamma}^0$.
3. If a is finite, then $e^{\bar{F}^{-1}(t)} \in RV_{-a}^0$.

Lemma 1 ensures that when F is well-behaved, (15) satisfies the conditions imposed in Theorem 3 for a wide range of parameter values. The following proposition is then an immediate implication of Theorem 3 and Lemma 1.

Proposition 3. *Let F be well behaved with tail index γ . Let $j, \psi \geq 0, k \geq 1$ and let $x^j e^{\psi x} f^k(x)$ be $[w_l, w_u)$ -integrable. If $j > 0$, assume that $w_u > 0$. If $\psi = 0$, we can treat $\psi a = 0$ in the*

²⁰To understand the correction factor, start with the linear case $G(x) = x$, in which case the theorem gives $\mathbb{E}[M_n] \sim \bar{F}^{-1}(1/n) \Gamma(-\gamma+1)$. Then the correction factor arises because the distribution of the maximum is $F^n(x)$, not $F(x)$. For distributions with an exponential type tail, $\gamma = 0$ and no correction is required. For distributions with a power type tail and finite mean, $\gamma \in (0, 1)$, an upward correction is needed. To provide some intuition for this, consider the $\log(-\log \mathbb{P}\{M_n \leq t\})$, and where the distribution F is either Gumbel or Fréchet, see Table 4. In case of the Gumbel one finds $\log n - t$, while the Fréchet gives $\log n - \alpha \log t$. Take n and t large. In the Gumbel case n plays a minor role, while in the case of the distribution n and t are of similar order of magnitude, so that n affects the distribution and its moments. More generally, if $G(x)$ is not linear, the tail behavior of $G(x)$ interacts with the tail behavior of $F(x)$. Both functions then determine ρ in the correction factor as indicated in the theorem. For example, take $G(x) = x^m$ and $F(x) = 1 - x^{-\alpha}$, $m < \alpha$, then $\mathbb{E}[(M_n)^m] \simeq n^{m/\alpha} \Gamma(1 - m/\alpha)$.

following expressions. If $(k - j - 1)\gamma - \psi a + k > 0$, then as $n \rightarrow \infty$,

$$\int_{w_l}^{w_u} x^j e^{\psi x} f^k(x) F(x)^{n-l} dx \sim \begin{cases} n^{-1} \left(\bar{F}^{-1}(1/n)\right)^j e^{\psi \bar{F}^{-1}(1/n)} f^{k-1}\left(\bar{F}^{-1}(1/n)\right) \Gamma((k - j - 1)\gamma - \psi a + k) : w_u = \infty \\ n^{-1} w_u^j e^{\psi w_u} f^{k-1}\left(\bar{F}^{-1}(1/n)\right) \Gamma((k - 1)\gamma + k) : w_u < \infty \end{cases}.$$

Proposition 3 allows us to approximate (15) for well-behaved distributions.²¹ The parameter restriction $(k - j - 1)\gamma - \psi a + k > 0$ is necessary to ensure that (15) does not diverge. For our purposes, this restriction is rather mild. One notable exception is that when $\psi > 0$, we cannot analyze heavy-tailed distributions (which have fatter-than-exponential tails) such as the lognormal distribution; for these distributions, $a = \infty$.²²

In fact, Theorem 1 is now an immediate corollary of Proposition 3. More generally, these results are relatively easy to apply. For example, the key mathematical objects in Theorem 1, γ and $n f\left(\bar{F}^{-1}(1/n)\right)$, are easy to calculate for most distributions of interest, and are listed for commonly used distributions in Table 4. The following fact, which is verified using Lemma A1.6, may often be useful to simplify calculations further for the case $\gamma \neq 0$: as $n \rightarrow \infty$,

$$\frac{1}{n f\left(\bar{F}^{-1}(1/n)\right)} \sim \begin{cases} \gamma \bar{F}^{-1}(1/n), & \gamma > 0 \\ -\gamma(w_u - \bar{F}^{-1}(1/n)), & \gamma < 0 \end{cases}.$$

5 Conclusion

The choice of noise distributions in random demand models is often influenced by tractability concerns. It is important to understand the consequences of these modeling choices and, when possible, to expand the set of tractable models. With this challenge in mind, our paper makes three sets of contributions.

First, we derive equilibrium markups for general noise distributions in various types of monopolistically competitive markets. We show that markups are asymptotically determined by the tail behavior of the distribution of taste shocks.

²¹For an antecedent to this result, see Maller and Resnick (1984).

²²Here we define a distribution to be heavy-tailed if $e^{\lambda x} \bar{F}(x) \rightarrow \infty$ as $x \rightarrow \infty$ for all $\lambda > 0$. To see why $a = \infty$ in this case, note that $\lim_{x \rightarrow \infty} \bar{F}(x)/f(x) = \infty$ implies $-\frac{d}{dx} \log \bar{F}(x) = o(1)$ as $x \rightarrow \infty$, so $-\log \bar{F}(x) = o(x)$ and $e^{-\lambda x} = o(\bar{F}(x))$ for all λ .

Table 4: **Properties of Common Densities**

Densities are listed in order of increasing tail fatness whenever possible.

Name of distribution	f	γ	$nf\left(\overline{F}^{-1}(1/n)\right)$	$\overline{F}^{-1}(1/n)$
Uniform	$1, x \in [-1, 0]$	-1	n	$-\frac{1}{n}$
Bounded Power Law	$\alpha(-x)^{\alpha-1}, \alpha \geq 1, x \in [-1, 0]$	$-1/\alpha$	$\alpha n^{1/\alpha}$	$-n^{-1/\alpha}$
Weibull	$\alpha(-x)^{\alpha-1}e^{-(-x)^\alpha}, \alpha \geq 1, x < 0$	$-1/\alpha$	$\alpha n^{1/\alpha}$	$\sim -n^{-1/\alpha}$
Gaussian	$(2\pi)^{-1/2}e^{-x^2/2}$	0	$\sim \sqrt{2 \ln n}$	$\sim \sqrt{2 \ln n}$
Rootzen Class	$\kappa \lambda \phi x^{\alpha+\phi-1}e^{-x^\phi}, x > 0, \phi > 1$	0	$\sim \phi \lambda^{1/\phi} (\ln n)^{1-1/\phi}$	$\sim (\ln n)^{1/\phi}$
Gumbel	$\exp(-e^{-x} - x)$	0	~ 1	$\sim \ln n$
Exponential	$e^{-x}, x > 0$	0	1	$\ln n$
Log-normal	$(2\pi)^{-1/2}x^{-1}e^{-(\log^2 x)/2}, x > 0$	0	$\sim \frac{\sqrt{2 \ln n}}{\overline{F}^{-1}(1/n)}$	$\sim e^{\sqrt{2 \ln n}}$
Power law	$\alpha x^{-\alpha-1}, \alpha > 0, x \geq 1$	$1/\alpha$	$\alpha n^{-1/\alpha}$	$n^{1/\alpha}$
Fréchet	$\alpha x^{-\alpha-1}e^{-x^{-\alpha}}, \alpha > 0, x \geq 0$	$1/\alpha$	$\alpha n^{-1/\alpha}$	$\sim n^{1/\alpha}$

Second, our results reveal a substantial degree of “detail-independence.” Specifically, the behavior of price markups are asymptotically identical (up to a constant factor) for all models that we study. Moreover, for the wide class of distributions with a zero extreme value tail exponent – including the canonical case of Gaussian noise – we show that the elasticity of markups to the number of firms is asymptotically zero. In other words, for many types of large markets, markups are relatively insensitive to the degree of competition.

Third, we show how to approximate an integral that is useful for studying a wide range of economic environments in which extreme outcomes determine the equilibrium allocation. For example, our framework can be used to model imperfect competition in large-economies, including applications in macroeconomics and trade.

In conclusion, this paper uses extreme value theory to clarify the quantitative impact of competition on prices in the symmetric-firms case. We anticipate that many of our results will extend with quantitatively modest adjustments to the non-symmetric case. Extending our analysis to such cases poses an important technical problem for future research. Non-symmetries will introduce substantial additional mathematical challenges.

Appendix A Proofs

This appendix proves the methodological results from Section 4, then applies them to prove the economic results of Sections 2 and 3. To clarify notation: denote $f_n \sim g_n$ if $f_n/g_n \rightarrow 1$, $f_n = o(g_n)$ if $f_n/g_n \rightarrow 0$ and $f_n = O(g_n)$ if there exists $M > 0$ and $n' \geq 1$ such that for all $n \geq n'$, $|f_n| \leq M |g_n|$.

Methodological Results

We start by collecting some useful facts about regular variation; for background, see Resnick (1987) or Bingham et al. (1989).

Lemma A1.

1. If $g(t) \in RV_a^0$, then the limit $\lim_{t \rightarrow 0} g(xt)/g(t) = x^a$ holds locally uniformly (with respect to x) on $(0, \infty)$.
2. If $\lim_{x \rightarrow 0} h(x)/s(x) = 1$, $\lim_{x \rightarrow 0} s(x) = 0$ and $g(x) \in RV_\rho^0$, then $g(h(x)) \sim g(s(x))$.
3. If $g(t) \in RV_a^0$ and $h(t) \in RV_b^0$, then $g(t)h(t) \in RV_{a+b}^0$.
4. If $g(t) \in RV_a^0$, $h(t) \in RV_b^0$ and $\lim_{t \rightarrow 0} h(t) = 0$, then $g \circ h(t) \in RV_{ab}^0$.
5. If $g(t) \in RV_a^0$ and non-decreasing, then $g^{-1}(t) \in RV_{a-1}^0$ if $\lim_{t \rightarrow 0} g(t) = 0$.
6. Let $U \in RV_\rho^0$. If $\rho > -1$ (or $\rho = -1$ and $\int_0^x U(t) dt < \infty$), then $\int_0^x U(t) dt \in RV_{\rho+1}^0$ and

$$\lim_{x \rightarrow 0} \frac{xU(x)}{\int_0^x U(t) dt} = \rho + 1.$$

If $\rho \leq -1$, then for $\bar{x} > 0$, $\int_x^{\bar{x}} U(t) dt \in RV_{\rho+1}^0$ and

$$\lim_{x \rightarrow 0} \frac{xU(x)}{\int_x^{\bar{x}} U(t) dt} = -\rho - 1.$$

7. If $\lim_{t \rightarrow \infty} tj'(t)/j(t) = \rho$, then $j \in RV_\rho^\infty$. Similarly, if $\lim_{t \rightarrow 0} tj'(t)/j(t) = \rho$, then $j \in RV_\rho^0$.
8. If $g \in RV_\rho^\infty$ and $\varepsilon > 0$, then $g(t) = o(t^{\rho+\varepsilon})$ and $t^{\rho-\varepsilon} = o(g(t))$ as $t \rightarrow \infty$; and if $g \in RV_\rho^0$ and $\varepsilon > 0$, then $g(t) = o(t^{\rho-\varepsilon})$ and $t^{\rho+\varepsilon} = o(g(t))$ as $t \rightarrow 0$.

Proof. See the online appendix. □

Our proof of Theorem 3 depends critically on the following result.

Theorem A1. (*Karamata's Tauberian Theorem*) Assume $U : (0, \infty) \rightarrow [0, \infty)$ is weakly increasing, $U(x) = 0$ for $x < 0$, and assume $\int_0^\infty e^{-sx} dU(x) < \infty$ for all sufficiently large s . With $\alpha \geq 0$, $U(x) \in RV_\alpha^0$ if and only if

$$\int_0^\infty e^{-sx} dU(x) \sim_{s \rightarrow \infty} U(1/s) \Gamma(\alpha + 1).$$

For a proof, see Bingham et al. (1987, pp.38, Th. 1.7.1') or Feller (1971, XIII.5, Th. 1) for another version of Karamata's Tauberian theorem.

Proof of Theorem 3. Assume for now that $G(x) \geq 0$ for all $x \in (w_l, w_u)$; we relax this assumption later. Differentiation of $\mathbb{P}(M_n \leq x) = F^n(x)$ gives the density of M_n : $f_n(x) = nf(x)F^{n-1}(x)$. Using the change of variable $x = \bar{F}^{-1}(t)$ and observing that $d\bar{F}^{-1}(t)/dt = -1/f(\bar{F}^{-1}(t))$

$$\begin{aligned} \mathbb{E}[G(M_n)] &= \int_{w_l}^{w_u} G(x)nf(x)F^{n-1}(x)dx \\ &= n \int_{w_l}^{w_u} G(x)F^{n-1}(x)(f(x)dx) \\ &= n \int_0^1 G(\bar{F}^{-1}(t))[F(\bar{F}^{-1}(t))]^{n-1}dt \\ &= n \int_0^1 \widehat{G}(t)(1-t)^{n-1}dt. \end{aligned}$$

We next use the change in variables $x = -\ln(1-t)$, so $t = 1 - e^{-x}$, $dt = e^{-x}dx$, and so

$$\mathbb{E}[G(M_n)] = n \int_0^\infty \widehat{G}(1 - e^{-x}) e^{-x} e^{-n'x} dx$$

where $n' = n - 1$. Define $h(x) = \widehat{G}(1 - e^{-x}) e^{-x}$, and $\mu(x) = \int_0^x h(y) dy$. Since \widehat{G} is regularly varying at zero with index $\rho > -1$, Lemma A1.8 implies that $\int_0^s \widehat{G}(t) dt < \infty$ for sufficiently small s . This, with the assumptions $G(t) \geq 0$ and $\int_s^1 \widehat{G}(t) dt < \infty$ for all $s \in (0, 1)$, ensure that $\mu(x) = \int_0^{1-e^{-x}} \widehat{G}(t) dt$ is finite and non-decreasing on $[0, \infty)$. By Lemma A1.2,

$h(x) \sim_{x \rightarrow 0} \widehat{G}(x)$. So $h \in RV_\rho^0$, and by Lemma A1.6

$$\mu(x) = \int_0^x h(y) dy \sim_{x \rightarrow 0} \frac{1}{1+\rho} h(x)x \sim_{x \rightarrow 0} \frac{1}{1+\rho} \widehat{G}(x)x.$$

Therefore, $\mu(x) \in RV_{\rho+1}^0$. Noting our assumption that $\rho+1 > 0$, we can now apply Karamata's Theorem A1 in combination with the last expression to obtain

$$\begin{aligned} \int_0^\infty e^{-n'x} d\mu(x) &\sim_{n' \rightarrow \infty} \mu(1/n') \Gamma(2+\rho) \\ &\sim_{n' \rightarrow \infty} \frac{1}{1+\rho} \widehat{G}(1/n') (n')^{-1} \Gamma(2+\rho) \\ &\sim_{n \rightarrow \infty} \widehat{G}(1/n) n^{-1} \Gamma(1+\rho). \end{aligned}$$

Thus

$$\begin{aligned} \mathbb{E}[G(M_n)] &= n \int_0^\infty e^{-n'x} d\mu(x) \\ &\sim n \widehat{G}(1/n) n^{-1} \Gamma(1+\rho) = G(\overline{F}^{-1}(1/n)) \Gamma(1+\rho) \end{aligned}$$

holds when $G(x) \geq 0$ for all $x \in (w_l, w_u)$. Now relax the assumption that $G(x) \geq 0$ for all $x \in (w_l, w_u)$. Choose $\bar{t} \in (0, 1)$ such that $G(t) > 0$ for $t \in [0, \bar{t}]$. The assumption that $G(\cdot)$ is strictly positive in a left neighborhood of w_u ensures that such \bar{t} exists. Thus we can write

$$\mathbb{E}[G(M_n)] = n \int_0^{\bar{t}} \widehat{G}(t) (1-t)^{n-1} dt + n \int_{\bar{t}}^1 \widehat{G}(t) (1-t)^{n-1} dt$$

Consider $\widetilde{G} : (0, 1) \rightarrow \mathbb{R}$ defined by

$$\widetilde{G}(t) \equiv \begin{cases} \widehat{G}(t) & : t \leq \bar{t} \\ 0 & : t > \bar{t} \end{cases}.$$

It is easy to check that \widetilde{G} satisfies the conditions of the theorem and additionally is weakly positive everywhere on (w_l, w_u) . The argument above shows that as $1/n \rightarrow 0$

$$n \int_0^{\bar{t}} \widehat{G}(t) (1-t)^{n-1} dt = n \int_0^1 \widetilde{G}(t) (1-t)^{n-1} dt \sim \widetilde{G}(1/n) \Gamma(1+\rho) \sim \widehat{G}(1/n) \Gamma(1+\rho). \quad (20)$$

To complete the proof we demonstrate that as $n \rightarrow \infty$

$$\left| \int_{\bar{t}}^1 \widehat{G}(t) (1-t)^{n-1} dt \right| = o \left(\int_0^{\bar{t}} \widehat{G}(t) (1-t)^{n-1} dt \right).$$

First, by (20): for $n \rightarrow \infty$,

$$\int_0^{\bar{t}} \widehat{G}(t) (1-t)^{n-1} dt \sim n^{-1} \widehat{G}(1/n) \Gamma(1+\rho) \in RV_{-\rho-1}^\infty.$$

Lemma A1.8 implies that $\int_0^{\bar{t}} \widehat{G}(t) (1-t)^{n-1} dt > n^{-\rho-1-\varepsilon}$ for sufficiently large n and given some $\varepsilon > 0$. Also,

$$\begin{aligned} \left| \int_{\bar{t}}^1 \widehat{G}(t) (1-t)^{n-1} dt \right| &\leq \int_{\bar{t}}^1 |\widehat{G}(t)| (1-t)^{n-1} dt \\ &\leq (1-\bar{t})^{n-1} \int_{\bar{t}}^1 |\widehat{G}(t)| dt \\ &\leq (1-\bar{t})^{n-1} \int_0^1 |\widehat{G}(t)| dt. \end{aligned}$$

By assumption $\int_s^1 |\widehat{G}(t)| dt < \infty$ for all $s \in (0, 1)$, therefore

$$\frac{\left| \int_{\bar{t}}^1 \widehat{G}(t) (1-t)^{n-1} dt \right|}{\int_0^{\bar{t}} \widehat{G}(t) (1-t)^{n-1} dt} \leq \frac{(1-\bar{t})^{n-1} \int_0^1 |\widehat{G}(t)| dt}{n^{-\rho-1-\varepsilon}} = o(1) \text{ as } n \rightarrow \infty.$$

This completes the proof. □

Proof of Lemma 1. See the online appendix. □

Proof of Proposition 3. Follows immediately from Theorem 3 and Lemma 1. □

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Proof of Theorem 1. Follows immediately from Proposition 3. □

Proof of Proposition 1. Treating n as continuous, we have

$$\frac{n}{\mu_n^{PS}} \frac{d\mu_n^{PS}}{dn} = - \left(\frac{2n-1}{n-1} + \frac{n \int f^2(x) F^{n-2}(x) \log F(x) dx}{\int f^2(x) F^{n-2}(x) dx} \right).$$

Noting that $-\log(1-x) \sim x \in RV_1^0$, applying Theorem 3 to $G(x) \equiv \frac{f(x)}{F(x)} \log F(x)$, using Lemma A1.3, we obtain

$$\int f^2(x) F^{n-2}(x) \log F(x) dx \sim -n^{-2} f(U_n) \Gamma(3 + \gamma).$$

Together with Theorem 1, it follows that

$$\frac{n}{\mu_n} \frac{d\mu_n}{dn} = - \left(2 - \frac{n^{-2} n f(\bar{F}^{-1}(1/n)) \Gamma(3 + \gamma)}{n^{-2} n f(\bar{F}^{-1}(1/n)) \Gamma(2 + \gamma)} + o(1) \right) = \gamma + o(1). \quad \square$$

Proof of Proposition 2. See the online appendix. □

Proof of Theorem 2. The Sattinger case follows directly from Proposition 3. For the Hart case, see the online appendix. □

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