



Moe:

“This thing can **flash fry** a **Water Buffalo** in **40 seconds.**”



Homer:

“Ohhhhh, **40 seconds!**
But I want mine Now.”

Jerry Seinfeld

“I never get enough sleep. I stay up late at night, cause I’m Night Guy. Night Guy wants to stay up late. ‘What about getting up after five hours sleep?’, oh that’s Morning Guy’s problem. That’s not my problem, I’m Night Guy. I stay up as late as I want. So you get up in the morning, you’re exhausted, groggy... oooh I hate that Night Guy! See, Night Guy always screws Morning Guy. There’s nothing Morning Guy can do.”

Intertemporal Choice Theory

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Outline for this lecture:

1. Introduction: thought experiments
2. Discount functions and discount rates
3. Quasi-hyperbolic discount function
4. Dynamic inconsistency
5. Naifs, Sophisticates, and Partial Naifs
6. Dynamic Programming: Search
7. Dynamic Programming: Consumption
8. Continuous time implementation

Next lecture: Empirical evidence

1 Introduction: motivational observations

Check the item below that you chose 'today':

1 wake up early to work on thesis

2 sleep in

Check the item below that you plan/intend to choose next week:

1' wake up early to work on thesis

2' sleep in

Some plans you don't often hear:

I plan to watch more TV next year.

I plan to eat more donuts next year.

I plan to use more crack next year.

I plan to smoke more cigarettes next year.

I plan to borrow more on my credit card next year.

I plan to exercise less next year.

I plan to wake up later next year.

Are self-reports trustworthy?

And even if they are trustworthy, what do they reflect?

Immediacy seems to be implicated in these behaviors.

Plans, forecasts, and choices made at a distance tend to reflect more patience than choices made in the present.

- I plan to study tomorrow
- I anticipate that I'll study tomorrow
- I commit to study tomorrow

Today I'll watch Hulu videos

2 Discount functions and rates

- Discount function: $D(\tau)$
- u **utils** in τ periods are psychologically worth $D(\tau)u$ utils today
- This is **not** the rate of intertemporal transformation – the interest rate
- Discount rate: rate of decline in the discount function

$$\rho(\tau) \equiv - \frac{dD(\tau)/d\tau}{D(\tau)}$$

– rate at which value of a util declines with delay

- Exponential discounting: $D(\tau) = \delta^\tau$

- For exponential case

$$-\frac{dD(\tau)/d\tau}{D(\tau)} = -\ln \delta = \rho \simeq 1 - \delta$$

- Exponential discount functions imply that discount rates do not change with horizon

$$\rho = \rho(\tau) \equiv -\frac{dD(\tau)/d\tau}{D(\tau)}$$

Calibrating exponential discounting:

If we discount utils tomorrow by 1%, then the one-year discount factor is 0.99^{365} .

- 100 utils in a year are worth 2.6 utils today.
- 100 utils in 10 years are worth 1×10^{-14} utils today.

If we discount utils in a year by 5%, then the one-day discount factor is $0.95^{1/365}$.

- 100 utils tomorrow are worth 99.99 utils today.

Some alternatives to exponential discounting (Samuelson, Strotz)

$$\frac{1}{t} \quad \text{Herrnstein and Ainslie (1960's)}$$

$$\frac{1}{1 + kt} \quad \text{Mazur (1970's)}$$

$$(1 + \alpha t)^{-\beta/\alpha} \quad \text{Loewenstein and Prelec (1990's)}$$

Discount rate is monotonically declining with horizon.

3 Quasi-hyperbolic discounting

- aka hyperbolic, present-biased, " β, δ ", quasi-geometric
- Discount rates are higher in the short-run than in the long-run.
- More impatience trading off **utils today** vs. tomorrow than trading off **utils** on day 100 vs. day 101.
- In other words, subjects have a higher short-run discount rate (**today** vs. tomorrow) than their long-run discount rate (day 100 vs day 101).

- The *quasi-hyperbolic discount function* (Phelps and Pollak 1968, Akerlof 1992, Laibson 1997, O'Donoghue and Rabin 1999):

$$D(\tau) = \begin{cases} 1 & \text{if } \tau = 0 \\ \beta \cdot \delta^\tau & \text{if } \tau \in \{1, 2, \dots\} \end{cases} .$$

- We then can write the utility function as,

$$\begin{aligned} U_t &= u_t + \beta\delta u_{t+1} + \beta\delta^2 u_{t+2} + \beta\delta^3 u_{t+3} + \dots \\ &= u_t + \beta \left(\delta u_{t+1} + \delta^2 u_{t+2} + \delta^3 u_{t+3} + \dots \right) \end{aligned}$$

- We tend to think that $\beta \ll 1$ and $\delta < 1$.
- E.g., $\beta = 2/3$ (one day) and $\delta \approx 1$.

- Consider a special case to build intuition: $\beta = \frac{1}{2}$ and $\delta \simeq 1$.

$$D(\tau) = \{1, \beta\delta, \beta\delta^2, \beta\delta^3, \dots\} = \left\{1, \frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \dots\right\}$$

- Intuition: relative to the present period, all future periods are worth less (weight $\frac{1}{2}$).
- All of the discounting takes place between the present and the immediate future.
- In the 'long-run' we're relatively patient — utils tomorrow are just as valuable as utils the day after tomorrow.

Model implies that decisions are sensitive to the timing of benefits and costs.

Are the **utility** benefits and **utility** costs **yoked** (i.e., together) in time, or is one in the present and the other in the future?

Is the playing field level (yoked case) when the agent makes a decision, OR, is there a present/future wedge?

For the examples that follow, we'll assume that $\beta = 1/2$ and $\delta = 1$.

Are the **utility** benefits (B) and costs (C) **yoked** in time, or is one in the present and the other in the future?

- Planning to eat free candy tomorrow. (Yoked benefits and costs.)

$$\beta (B_{PLEASURE} - C_{HEALTH}) = \frac{1}{2} (2 - 3) < 0$$

- Eating free candy today. (Unyoked benefits and costs.)

$$B_{PLEASURE} + \beta (-C_{HEALTH}) = 2 + \frac{1}{2} (-3) > 0$$

See Wertenbroch (1998)

Future Candy (yoked costs and benefits)

	Costs	Benefits
Now	0	0
Later	$C_{HEALTH} = 3$	$B_{PLEASURE} = 2$

Present Candy (unyoked costs and benefits)

	Costs	Benefits
Now	0	$B_{PLEASURE} = 2$
Later	$C_{HEALTH} = 3$	0

Are the **utility** benefits and costs yoked in time, or is one in the present and the other in the future?

- Buying future People magazines by subscribing. (Yoked costs and benefits)

$$\beta (B_{PLEASURE} - C_{\$}) = \frac{1}{2} (2 - 3) < 0$$

- Buying present People magazine from check-out line. (Unyoked costs and benefits)

$$B_{PLEASURE} + \beta (-C_{\$}) = 2 + \frac{1}{2} (-3) > 0$$

See Oster and Scott-Morton (2006)

Future *People Magazine* (yoked costs and benefits)

	Costs	Benefits
Now	0	0
Later	$C_{\$} = 3$	$B_{PLEASURE} = 2$

Present *People Magazine* (unyoked costs and benefits)

	Costs	Benefits
Now	0	$B_{PLEASURE} = 2$
Later	$C_{\$} = 3$	0

Remark: The financial cost is later, since we assume that current expenditure crowds out later expenditure. This is a natural consequence of the dynamic budget constraint.

Are the **utility** benefits and costs yoked in time, or is one in the present and the other in the future?

- Order future foreign movie instead of future fun movie. (Yoked.)

$$\beta (B_{KNOWLEDGE} - C_{FUN}) = \frac{1}{2} (3 - 2) > 0$$

- Watch present foreign movie instead of present fun movie? (Unyoked.)

$$-C_{FUN} + \beta (B_{KNOWLEDGE}) = -2 + \frac{1}{2} (3) < 0$$

See Milkman et al (2010).



Figure 1: Canonical foreign movie. A man seeks answers about life, death, and the existence of God as he plays chess against the Grim Reaper during the Black Plague.

Future foreign movie instead of future fun movie (yoked costs and benefits)

	Costs	Benefits
Now	0	0
Later	$C_{FUN} = 2$	$B_{KNOWLEDGE} = 3$

Present foreign movie instead of present fun movie (unyoked)

	Costs	Benefits
Now	$C_{FUN} = 2$	0
Later	0	$B_{KNOWLEDGE} = 3$

The details matter.

- Build up \$5000 of debt on a credit card at 20% interest? Yes.
- Take out a home equity loan at 5% interest requiring three hours of paperwork and a two week processing delay? I'll do it next week.
- Take out a home equity loan at 10% interest – 'Preapproved with No Paperwork Required'? Yes.
- But what if the potential borrower who receives the 10% offer knows that 5% loans are available at another bank?

The details matter.

- Buy a new car, making \$2000 down-payment? No.
- Buy a new car, paying more interest, but making no downpayment (i.e., “\$2000 cash back”)? Yes.

3.1 Classes of experimental evidence (next lecture):

- Dynamic **money** choices: \$ at t or more \$ at $t + \tau$
 - I refer to them as MEL experiments: Money Earlier or Later
 - These types of experiments shouldn't measure the utility discount rate
 - In principle, they *should* measure the interest rate (arbitrage argument reviewed in next lecture)
 - But in practice, they don't seem to measure that either (see Rubinstein 1988, 2003 for 'similarity' interpretation of MEL experiments)

- Dynamic **consumption** choices: ice cream at t or more ice cream at $t + \tau$
 - These experiments are also confounded (e.g., trust, utility function curvature, subtle forms of intertemporal arbitrage), though they are better than money experiments for measuring time preferences
- Static choices (chocolate at t or orange at t)

3.2 Example of inferences from static choices:

Read and van Leeuwen (1998)

- Choose a snack now to eat next week:
 - overwhelming majority choose healthy (e.g., low calorie)
- Choose a snack to eat now:
 - overwhelming majority choose unhealthy (e.g., high calorie)

Illustrative model:

Let c represent calories (or cocaine, candy, cigarettes), with u strictly concave and v strictly convex.

$$\begin{aligned} U(c_{t-1}, c_t, c_{t+1}, \dots) = & [u(c_t) - v(c_{t-1})] \\ & + \beta\delta [u(c_{t+1}) - v(c_t)] \\ & + \beta\delta^2 [u(c_{t+2}) - v(c_{t+1})] \end{aligned}$$

Optimal level of consumption from the perspective of date t :

$$\begin{aligned} u'(c_t) &= \beta\delta v'(c_t) \\ u'(c_{t+1}) &= \delta v'(c_{t+1}) \end{aligned}$$

$$\beta = 1 \iff c_t = c_{t+1}$$

$$\beta < 1 \iff c_t > c_{t+1}$$

Formal derivation:

First, note that if $\beta = 1$, then $c_t = c_{t+1}$. Then note that β does not influence c_{t+1} . Finally, note that

$$u''(c_t) dc_t = d\beta \delta v'(c_t) + \beta \delta v''(c_t) dc_t.$$

Hence,

$$\frac{dc_t}{d\beta} = \frac{\delta v'(c_t)}{u'' - \beta \delta v''} < 0$$

It follows that $c_t > c_{t+1}$, implies $\beta < 1$.

4 Dynamic Inconsistency:

- Exercise has benefit today of -6. Exercise has delayed benefit of 8.
- Exercise today? No.

$$-6 + \frac{1}{2}(8) = -2.$$

- Exercise tomorrow? Yes.

$$0 + \frac{1}{2}(-6 + 8) = 1.$$

- But tomorrow you'll again want to postpone action (Akerlof 1992; O'Donoghue and Rabin 1999; Dellavigna and Malmendier 2004, 2006)

- *Dynamic consistency* means that early selves and later selves agree.
- In other words, optimal **contingent** plans do not change over time.
- So we can simply maximize at the beginning of time without worrying about later selves overturning the decisions of early selves.
- Exponential discounting (with a fixed utility function) is sufficient but not necessary for dynamic consistency.

- In many domains, early and late selves don't seem to agree.
- Preferences are dynamically inconsistent iff optimal **contingent** plans change over time.
- Some familiar resolutions. Does data like this have a role in economics?

Next month, I'll quit smoking.

Next week, I'll catch up on the required reading.

Tomorrow morning, I'll wake up early and exercise.

On New Year's Day, I'll start eating better.

Next weekend, I'll send in this rebate form.

Next month, I'll join the savings plan.

Early selves plan to “be good” (get up at 7AM to finish problem set)

Later self wants “instant gratification” (keep hitting snooze button)

When discount functions are not exponential, the intertemporal choice model generates a conflict between early and late selves: dynamic inconsistency.

Dynamically inconsistent model predicts “self-control problems” like procrastination, “laziness”, addiction, etc...

5 Naifs and Sophisticates

- Naifs falsely believe that future selves will maximize today's preferences (Strotz 1957).
 - Solution concept: maximization (mispredict future discount rates).
 - Prediction: never exercise (but join gym).
- Sophisticates have rational expectations (Strotz 1957)..
 - Solution concept: subgame perfect equilibrium.
 - Prediction: never exercise (and don't join gym).

- Partial naivite (O'Donoghue and Rabin, 2001)
 - Solution concept: subgame perfect equilibrium, assuming that all future selves use $\hat{\beta}$ with

$$\beta < \hat{\beta} < 1.$$

- Note that naifs use $\hat{\beta} = 1$ and sophisticates use $\hat{\beta} = \beta$.

Related issues to think about (conjectures):

- People often make the error of over-predicting productivity.
- People appear to be better at predicting other people's procrastination than their own procrastination.
- Predictions tend to be heavily influenced by domain-sensitive experience.

5.1 Extreme Implications (O'Donoghue and Rabin 1999)

5.1.1 Naives

- Consider a naif with $\beta = \frac{1}{2}$ and $\delta = 1$.
- The naif has to finish a project by deadline T .
- In time period t , the (undiscounted) project costs $\left(\frac{3}{2}\right)^t$ utils to execute.
- When will the naif do the project?

From the current self's perspective, it's always better to postpone doing the project until next period:

$$\begin{aligned}\left(\frac{3}{2}\right)^t &> \beta\delta \left(\frac{3}{2}\right)^{t+1} \\ &= \frac{1}{2} \left(\frac{3}{2}\right)^{t+1} \\ &= \frac{3}{4} \left(\frac{3}{2}\right)^t\end{aligned}$$

When will the project be completed?

(Partial naives can make the same kind of mistakes.)

5.1.2 Sophisticates:

Consider the same model as above.

When will a sophisticate do the project?

1. If T is even, then sophisticates will do the project in even periods (and not in odd periods).
2. If T is odd, then sophisticates will do the project in odd periods (and not in even periods).

However, situation gets less bizarre when you add uncertainty (below).

Another problem with the sophisticated (and partially naive) model:

- We see little endogenous commitment for commitment's sake.
- Most commitment is ancillary (e.g., obligatory monthly mortgage payments).
- Very little commitment is gratuitous and advertised as such (e.g., Christmas clubs).
- One exception: StickK. But this is the brain child of Dean Karlan and Ian Ayres, behavioral economists who have worked on hyperbolic discounting, so this doesn't really count.

6 Dynamic Programming: Search and Procrastination

- See Akerlof (1992) and O'Donoghue and Rabin (1999) for early papers on Procrastination
- Today: Carroll et al (2010)
- $0 < \beta < 1$; $\delta = 1$ (interpretation: daily model so $\delta \simeq 1$)
- Per period loss from delay L (e.g., lost per-period benefit)
- Stochastic action cost c_t drawn from a uniform distribution on the interval $[\underline{c}, \bar{c}]$

6.1 Sophisticates

Let W represent the current cost function

$$W(c) = \begin{cases} c & \text{if act} \\ \beta [L + EV(c')] & \text{if wait} \end{cases} \quad (1)$$

Let V represent the “exponentially discounted” continuation cost function

$$V(c) = \begin{cases} c & \text{if act tomorrow} \\ L + EV(c') & \text{if wait tomorrow} \end{cases} \quad (2)$$

Study stationary equilibrium: “cutoff threshold” c^* .

Agents must be indifferent in the current period between acting and waiting at the cutoff,

$$c^* = \beta [L + EV(c')]. \quad (3)$$

Our problem can be reduced to two equations

$$\begin{aligned} c^* &= \beta [L + EV] \\ EV &= \int_{c \leq c^*} c dF(c) + \int_{c > c^*} [L + EV] dF(c) \end{aligned}$$

and two unknowns: c^* and EV .

Proposition 6.1 *The equilibrium cutoff threshold is*

$$c^* = \frac{\underline{c} + \sqrt{\underline{c}^2 [1 - (2 - \beta)\beta] + 4\beta \left(1 - \frac{\beta}{2}\right) (\bar{c} - \underline{c}) L}}{2 - \beta}. \quad (4)$$

6.2 Properties of c^* :

- How does c^* change with L , the flow cost?
- How does c^* change with β , the short-term discount factor?
- Is c^* above \underline{c} ?
- Is c^* below \bar{c} ?

6.3 Procrastination with sophisticates

- Let c^{**} be the *desired* future threshold. So,

$$c^{**} \equiv c_{\beta=1}^* = \underline{c} + \sqrt{2(\bar{c} - \underline{c})L}. \quad (5)$$

- How does c^{**} compare with $c_{\beta < 1}^*$?
- What is the probability that an agent procrastinates in a given period?
- Illustrative calibration: $\underline{c} = 0$ and $\bar{c} = 1$.

$$c^{**} - c^* = \sqrt{2L} \left(1 - \sqrt{\frac{\beta/2}{1 - \beta/2}} \right)$$

- If $\sqrt{2L} = 1$, exponentials always do it. If, $\beta = 2/3$ probability of procrastination is

$$c^{**} - c^* = 1 - \sqrt{\frac{\beta/2}{1 - \beta/2}} = 1 - \frac{1}{\sqrt{2}} = 0.29.$$

6.4 Testing your intuition:

- Consider the boundary case in which $\bar{c} = \underline{c}$.
- What is the equilibrium action rule?

6.5 Procrastination with naives

- Let c_N^* be the equilibrium Naive threshold.
- Is c_N^* greater than or less than c^* ?
- Is c_N^* greater than or less than \underline{c} ?
- Is that the good news or the bad news?

7 Dynamic programming: stationary consumption problem

- Let c represent consumption
- Let x represent cash-on-hand
- Let \tilde{y} represent iid stochastic income
- Let R represent gross interest rate

$$x_{t+1} = R(x_t - c_t) + \tilde{y}_{t+1}$$

- A (Markov) strategy is a map from state x to control c .

- Let V be the continuation-value function, W be the current-value function and C be the consumption function. Then:

$$V(x) = U(C(x)) + \delta \mathbf{E}[V(R(x - C(x)) + y)]$$

$$W(x) = U(C(x)) + \beta\delta \mathbf{E}[V(R(x - C(x)) + y)]$$

$$C(x) = \underset{c}{\operatorname{argmax}} U(c) + \beta\delta \mathbf{E}[V(R(x - c) + y)]$$

- Envelope Theorem: $W'(x) = U'(C(x))$.
- First-order-condition: $U'(C(x)) = R\beta\delta \mathbf{E}[V'(R(x - C(x)) + y)]$.
- Identity linking V and W : $\beta V(x) = W(x) - (1 - \beta)U(C(x))$.

7.1 Problem is recursive

- Start with V .

- Find C :

$$C(x) = \operatorname{argmax}_c U(c) + \beta \delta \mathbf{E}[V(R(x - c) + y)].$$

- Find \hat{V} :

$$\hat{V}(x) = U(C(x)) + \delta \mathbf{E}[V(R(x - C(x)) + y)]$$

- In this way, generate an operator $T : V \mapsto \hat{V}$.

7.2 Generalized Euler Equation (Harris and Laibson 2001).

We have

$$\begin{aligned}u'(c_t) &= R\beta\delta \mathbf{E}_t[V'(x_{t+1})] \\&= R\delta \mathbf{E}_t\left[W'(x_{t+1}) - (1 - \beta)u'(c_{t+1})\frac{dC_{t+1}}{dx_{t+1}}\right] \\&= R\delta \mathbf{E}_t\left[u'(c_{t+1}) - (1 - \beta)u'(c_{t+1})\frac{dC_{t+1}}{dx_{t+1}}\right].\end{aligned}$$

Follows from FOC, differentiated identity, and envelope theorem. Simplifying,

$$u'(c_t) = R \mathbf{E}_t \left[\beta\delta \left(\frac{dC_{t+1}}{dX_{t+1}} \right) + \delta \left(1 - \frac{dC_{t+1}}{dX_{t+1}} \right) \right] u'(c_{t+1}).$$

Quasi-hyperbolics are highly patient when $\frac{dC_{t+1}}{dX_{t+1}} \simeq 0$, and highly impatient when $\frac{dC_{t+1}}{dX_{t+1}} \simeq 1$.

Calibration of steady state with no growth:

- $u(c) = \ln(c)$.
- In a standard exponential discounting model (i.e., $\beta = 1$), we have

$$\delta R = 1,$$

so the discount rate, $-\ln \delta$, is equal to the interest rate, $\ln R$.

- What happens in the quasi-hyperbolic economy?
- Suppose $\beta = 2/3$ and $\delta = 0.975$, what is the steady state interest rate?

- If λ is the APC=MPC, then in steady state,

$$\frac{1}{\lambda} = R[\beta\delta\lambda + \delta(1 - \lambda)] \frac{1}{(1 - \lambda)R\lambda}$$
$$1 = (1 - \lambda)R$$

- Calibrate $\beta = 2/3$, $\delta = 0.975$:

$$\lambda = \frac{1 - \delta}{1 - \delta(1 - \beta)} = 0.04.$$
$$r \simeq \lambda = 0.04.$$

- Why is the equilibrium interest rate so low?

8 Continuous time (Harris & Laibson 2013)

Transitions between (countable) selves occur at hazard rate λ .

Let τ_n be the (stochastic) duration of self n .

Each self values her next self discretely less than her present self, discounting it by the factor $0 < \beta \leq 1$.

All selves also discount exponentially with discount factor $0 < \delta < 1$. Let $\gamma = -\ln \delta$.

Discount function of self n .

$$D_n(t) = \left\{ \begin{array}{ll} \delta^t & \text{if } t \in [0, \tau_n) \\ \beta \delta^t & \text{if } t \in [\tau_n, \infty) \end{array} \right\}. \quad (6)$$

8.1 Application to a savings problem:

- Let c be consumption.
- Let y be labor income (non-collateralizable).
- Let x be cash on hand: $dx = (\mu x + y - c)dt + \sigma x dz$

- Current value function, $w(x)$:

$$\gamma w = u(c) + (\mu x + y - c) w' + \frac{1}{2} \sigma^2 x^2 w'' + \lambda (\beta v - w) \quad (7)$$

- Continuation value function, $v(x)$:

$$\gamma v = u(c) + (\mu x + y - c) v' + \frac{1}{2} \sigma^2 x^2 v'' \quad (8)$$

- Consumption:

$$u'(c) = w'(x).$$

8.2 The Instantaneous Gratification Model: $\lambda \rightarrow \infty$.

- First differential equation collapses to:

$$w = \beta v \quad (9)$$

- Continuation value function, $v(x)$:

$$\gamma v = u(c) + (\mu x + y - c) v' + \frac{1}{2} \sigma^2 x^2 v'' \quad (10)$$

- Consumption:

$$u'(c) = w'(x) = \beta v'(x)$$

- In essence, two-equation system collapses to one-equation system. (w is superfluous)

8.3 Properties:

- Unique equilibrium (Bellman-system equivalence to an optimization problem; however policy function is not equivalent).
- Consumption drops discontinuously when agents hit the liquidity constraint.
- Policy functions are smooth in the interior of the state space.
- Single welfare criterion. Recall that $w = \beta v$.

9 Other research directions

- Applications.
- Empirical testing, particularly field data (next lecture.)
- Individual differences (Chabris et al 2008; Rubin, Sapienza, and Zingales 2008; Benjamin and Shapiro 2006).
- Pathways from naivete to sophistication?
- Why does society prefer opaque commitment?

- Neuro evidence linking self-regulation to the dorsal-lateral pre-frontal cortex (McClure, Laibson, Loewenstein, and Cohen 2004; McClure, Ericson, Laibson, Loewenstein, and Cohen 2007;).
- What makes β fall below one? Context dependent β effects.
 - Visceral (Loewenstein, 1996)
 - Cues (Laibson, 2001), Generalized temptation (Gul and Pesendorfer, 2002)
- Integrative two-brain models by Shefrin and Thaler (1981), Bernheim and Rangel (2004), Fudenberg and Levine (2004), Benhabib and Bisin (2004), and O'Donoghue and Loewenstein (2008).