

Cognitive Biases

- ① Errors in Probabilistic Reasoning, continued \Rightarrow
 - ① LSN, NBLLN, and SampleSizeNeglect in Inference \Rightarrow
 - ② Base-Rate Neglect (and Confirmation Bias) \Rightarrow
- ② Challenging features of non-Bayesian models \Rightarrow
 - ① Prospective beliefs and dynamic inconsistency \Rightarrow
 - ② **Question-Dependent/Elicitation-Dependent Beliefs** \Rightarrow
- ③ Features of Bayesian Reasoning (that may not hold) \Rightarrow
 - ① Model-free tests of Bayesian reasoning \Rightarrow
- ④ **Measuring Beliefs** ... is obviously the right thing to do,
 - ① and obviously the future, and starting (too slowly) \Rightarrow
 - ② Do it *very* carefully (without nihilism or obstructionism) \Rightarrow
- ⑤ Shortcomings in social inference \Rightarrow
 - ① Inversion Neglect, Economics-Neglect, and Non-Structural Models \Rightarrow
 - ② Redundancy Neglect \Rightarrow
- ⑥ (Delayed until tomorrow, time permitting: Inattention (notice the missing adjective...))

Evidence on LSN, NBLLN, & SSN in Inference \Rightarrow

Beautiful experiment: Griffin and Tversky (1992). \Rightarrow

- “Imagine that you are spinning a coin, and recording how often the coin lands heads and how often the coin lands tails. \Rightarrow Unlike tossing, which (on average) yields an equal number of heads and tails, spinning a coin leads to a bias favoring one side or the other because of slight imperfections on the rim of the coin (and an uneven distribution of mass). \Rightarrow Now imagine that you know that this bias is $3/5$. It tends to land on one side 3 out of 5 times. \Rightarrow But you do not know if this bias is in favor of heads or in favor of tails.”

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Tempting model to capture both: **Sample-Size Neglect.**⇐

- Kahneman & Tversky (1973) and Griffin and Tversky (1992):⇐
 - People attend to proportions, not sample size.⇐
 - Intuitive model, **not far off.**

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Cognitive Biases

Two possible *i.i.d.* coins each with prob = .5: \Rightarrow

- $\pi(h|\cdot) = \frac{3}{5}$ coin and a $\pi(h|\cdot) = \frac{2}{5}$ coin \Rightarrow
- Observe a set of flips h, t . \Rightarrow
- Bayes' Law says $\frac{\pi(\theta|h,t)}{\pi(1-\theta|h,t)} \equiv l(h,t) = \left(\frac{3}{2}\right)^{h-t}$. \Rightarrow
- So Bayesian inference from (h,t) depends solely on $h - t$. \Rightarrow
- That's all well and good ... but: \Rightarrow
 - Only geeks think that way. \Rightarrow
 - And only when paying attention. \Rightarrow
 - People in fact base beliefs on how close $\frac{h}{h+t}$ looks to $\frac{3}{5}$ vs. $\frac{2}{5}$. \Rightarrow

A tale of two tables **presenting the same data**:

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Cognitive Biases

Over-infer from small samples, under-infer from large samples. \Leftarrow

Sample of (h,t)	$h + t$	$h - t$	Median $P(\theta = \frac{3}{5} h, t)$	Proper $B(\theta = \frac{3}{5} h, t)$
5,0	5	5	.92	.88
7,2	9	5	.77	.88
11,6	17	5	.64	.88
19,14	33	5	.60	.88
3,0	3	3	.85	.77
4,1	5	3	.80	.77
6,3	9	3	.67	.77
10,7	17	3	.60	.77
2,1	3	1	.63	.60
3,2	5	1	.60	.60
5,4	9	1	.55	.60
9,8	17	1	.54	.60



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Meta-Lesson: **proportional thinking** \Rightarrow

Sample of (h,t)	% heads	Median $P(\theta = \frac{3}{5} h, t)$	Proper $B(\theta = \frac{3}{5} h, t)$
5,0	100%	.92	.88
3,0	100%	.85	.77
4,1	80%	.80	.77
7,2	78%	.77	.88
6,3	67%	.67	.77
2,1	67%	.63	.60
11,6	65%	.60	.88
3,2	60%	.60	.60
10,7	59%	.60	.77
19,14	58%	.60	.88
5,4	55%	.55	.60
9,8	53%	.54	.60



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- Basing on proportions leads to “Sample-Size Neglect” : \Rightarrow

Under-using sample size. \Rightarrow

- If double h and t , double $h - t$
- hence change a Bayesian's inference. \Rightarrow
- But you won't change $\frac{h}{h+t}$. \Rightarrow
 - So people infer same. \Rightarrow
- **Over-infer from small samples, under-infer from large samples.** \Rightarrow
- Discussion in BRR (2012) why don't model just as "SSN"
- **Note:** sometimes underinference, underconfidence

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More attitude problem!⇒

- Is thinking in terms of $h - t$ System 1 or System 2?⇒
- Is thinking in terms of $\frac{h}{h-t}$ System 1 or System 2?⇒
- And if we knew Sissy were using System X...⇒
 - (whichever is the bad one)⇒
 - Would we know how she uses sample size?⇒

Whatever its conceptual usefulness and neuro-truth of system thinking...⇒

- Severe danger of false consciousness.⇒
- And empirical complacency.⇒

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Cognitive Biases

- As we'll see repeatedly with non-Bayesian models, framing effects and inconsistencies of various sorts:⇒

NBLLN inherently implies “clumping” issues...⇒

- Suppose Barney reads a summary of 10,000 individuals' experiences in *Consumer Reports*. Then he asks a sequence of 5 friends about their experience driving a Volvo.⇒
- Inherent to NBLLN, it matters whether Barney processes these 10,005 signals as...⇒
- a single sample of size 10,005?⇒
 - BRR call such a Barney “pooling” ⇒
- as one group 10,000 signals and five groups of 1 signal each.⇒
 - BRR call such a Barney “acceptive” ⇒
- as 10,005 separate signals⇒
 - BRR call such a “Barney” “atomizing” .⇒

We'll return to these issues.⇒

- Now move on to next bias, the grand daddy/mother of all IDM ⇒

Cognitive Biases

Hyphens. \Rightarrow

- What do each of the following mean? \Rightarrow
 - “base-rate neglect” \Rightarrow
 - “base rate neglect” \Rightarrow
- Or an incomplete-information game vs. incomplete information game \Rightarrow
- Or an extra-marital affair vs. extra marital affair \Rightarrow
- Or (I wish I could Richard to understand the distinction) \Rightarrow
 - Rational-choice theory vs. rational choice theory \Rightarrow
- A-B C means a C that is [of] A-B \Rightarrow
- A B C means a B-C that is A. \Rightarrow

Back to less important stuff \Rightarrow

- Model (stated, estimated over the years):

\rightarrow

Cognitive Biases

Now: Base-Rate Neglect \Rightarrow

- Model (stated, estimated over the years): \Rightarrow
- Base-rate-neglect sufferer (**Saki**) believes:

$$p(\theta|s) = \frac{p(s|\theta)p(\theta)^\alpha}{\sum_{\theta' \in \Theta} p(s|\theta')p(\theta')^\alpha} \cdot \Rightarrow$$

- where $\alpha \in [0, 1)$. \Rightarrow Extreme $\alpha = 0$ Saki:

$$p(\theta|s) = \frac{p(s|\theta)}{\sum_{\theta' \in \Theta} p(s|\theta')} \cdot \Rightarrow$$

- Combine LSN, GF, and N BLLN with BRN: \Rightarrow

$$p(\theta|s) = \frac{\tilde{p}(s|\theta)p(\theta)^\alpha}{\sum_{\theta'} \tilde{p}(s|\theta')p(\theta')^\alpha}.$$

Cognitive Biases

The textbook example of BRN: \Rightarrow

- Test for disease is 90% accurate (symmetrically) \Rightarrow
- 5% of tested population have disease \Rightarrow

If test positive, probability of disease? \Rightarrow

- Stats text: tells you right answer (32%) \Rightarrow
- Psych articles: shows people give wrong answer (say, 90%) \Rightarrow

But ... \Rightarrow

- You might be left with wrong impression from BRN ...

\Rightarrow

Cognitive Biases

Test	Priors	Posteriors		Frequency	
		Tommy	Saki		
Positive	5%	32%	90%	14%	⇒
Negative	5%	< 1%	10%	86%	

- Yes, her beliefs have moved too much ⇒
 - Tommy beliefs moved (up) 27% or (down) 4%. ⇒
 - Saki's beliefs moved (up) 85% or (up) 5%. ⇒
- But, while Saki beliefs too extreme after positive result ... ⇒
 - (which is the main image of BRN) ⇒
 - But too **moderate** after negative result ⇒
- Turns out a general feature of BRN: ⇒
 - Beliefs move too much, but are too moderate ⇒
 - In dynamic, i.i.d. model, ergodic non-convergence.

Cognitive Biases

- In fact, a bizarre “moderation effect” : \Rightarrow
 - buried in definition, buried in literature \Rightarrow
 - Staring in face from Day 1, but we decided it was an expose-worthy realization ... \Rightarrow

Test	Priors	Posteriors		Frequency	
		Tommy	Saki		
Positive	5%	32%	90%	14%	\Rightarrow
Negative	5%	< 1%	10%	86%	

- After **negative** signal, her beliefs move **up**. \Rightarrow
- Wouldn't necessarily bet on that implication here. \Rightarrow
 - But it is an implication that the model must own ... \Rightarrow
 - Needn't always apologize for bad results, but must always 'own' them. \Rightarrow

Two formalizations of this “moderation effect” : \Leftrightarrow

- Suppose any degree of BRN, $\alpha < 1$. \Leftrightarrow
- Suppose 2 states, with p_t on one of the two states. Then: \Leftrightarrow
- \forall signals $\sigma_t \in (0, 1)$, $\exists p_t < 1$ s.t. $\ln\left(\frac{p_{t+1}}{1-p_{t+1}}\right) < \ln\left(\frac{p_t}{1-p_t}\right)$. \Leftrightarrow
 - For all signals, there exist (strong enough) priors such that the signal weakens belief in the hypothesis. \Leftrightarrow
 - Notice ... I haven't told you which hypothesis I am talking about. \Leftrightarrow
- For all $p_t > \frac{1}{2}$, $\exists \sigma_t > \frac{1}{2}$ such that $\ln\left(\frac{p_{t+1}}{1-p_{t+1}}\right) < \ln\left(\frac{p_t}{1-p_t}\right)$. \Leftrightarrow
 - For all prior beliefs in favor of hypothesis, there exists a (weak enough) signal in favor that weakens belief in the hypothesis. \Leftrightarrow

But maybe we want to own ... return to evidence

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Cognitive Biases

Griffin and Tversky (1992): \Rightarrow Subjects' median posterior for combinations of samples and priors about 60% vs. 40% coin. \Rightarrow

- **Priors 1:1** \Rightarrow

(h, t)	Bayesian posterior	Median subject posterior	
(5, 5)	.50	.50	
(6, 4)	.69	.60	\Rightarrow
(7, 3)	.84	.70	
(8, 2)	.92	.80	
(9, 1)	.996	.90	

- So we see some underinference for sample sizes of 10. \Rightarrow
- Nothing to do with base-rate neglect. \Rightarrow It is NBLLN.

\rightarrow

Priors 1:9 \Rightarrow

(h, t)	Bayesian	NBLLN without BRN	Median Subject	
(5, 5)	.10	.10	.22	
(6, 4)	.20	.14	.45	\Rightarrow
(7, 3)	.36	.21	.60	
(8, 2)	.55	.31	.80	
(9, 1)	.74	.50	.85	

- Last 2 lines: over-inference due to BRN—**overcoming NBLLN**. \Rightarrow
- Like vast majority of BRN experiments: \Rightarrow
 - Pitting priors against signal, signal 'wins' when it shouldn't. \Rightarrow
- But...



Cognitive Biases

Priors 9:1 \Rightarrow

(h, t)	Bayesian	NBLLN without BRN	Median Subject	
(5, 5)	.90	.90	.60	
(6, 4)	.95	.93	.70	\Rightarrow
(7, 3)	.98	.95	.85	
(8, 2)	.99	.97	.925	
(9, 1)	.996	.988	.985	

- One of *very few* experiments where data same direction as base rate \Rightarrow
- Note the effect of $5h/5t$, $6h/4t$, and $7h/3t$ on beliefs. \Rightarrow
- This "moderation effect" may be more important for economics than much-emphasized over-inference.



Cognitive Biases

Fleshing out model in dynamic contexts. \Rightarrow

- **When signals arrive sequentially**, cumulative updating? \Rightarrow
 - Combine all signals together, then combine with original base rate? \Rightarrow
 - Or sequentially update—each new signal generates new base rate? \Rightarrow
- We assume 2^{nd} —central to many results. \Rightarrow
 - (Among other things, treats ‘base rates’ as equivalent to priors? \Rightarrow
- So today’s posteriors are tomorrow’s priors. \Rightarrow Then dynamic implications: \Rightarrow

$$\ln \left(\frac{p(\theta|s_1, \dots, s_t)}{p(\theta'|s_1, \dots, s_t)} \right) = \ln \left(\frac{p(s_t|\theta)}{p(s_t|\theta')} \right) + \alpha \ln \left(\frac{p(s_{t-1}|\theta)}{p(s_{t-1}|\theta')} \right) + \dots + \alpha^t \ln \left(\frac{p(\theta)}{p(\theta')} \right) \Rightarrow$$

- Data observed in the recent past matters more than data from distant past (even though signals i.i.d.). \Rightarrow
 - So, obviously, beliefs from same information depends on order. \Rightarrow
 - Forever and ever. \Rightarrow
- As if current day’s temperature affects belief in global warming? \Rightarrow
 - (Yi, Johnson, and Zaval, 2011).

Implications: \Rightarrow

- Recency effect: more recent data matters more. \Rightarrow
- Long-run beliefs ergodic \Rightarrow
 - initial prior ceases to matter. \Rightarrow
 - *Support* of ergodic distribution independent of true state. \Rightarrow
 - Never become fully confident. \Rightarrow
- Moderation effect: strong priors always dampened. \Rightarrow
- Range of these beliefs independent of truth. \Rightarrow
 - But frequency of different beliefs does depend on it. \Rightarrow
- Interesting implications in reputation context. \Rightarrow
 - Perpetually gain and lose reputation \Rightarrow
 - No eventual decay.




Cognitive Biases

Let us observe about NBLLN and BRN ... \Rightarrow

- Along with inattention, and inversion neglect/disagreement neglect/cursedness, discussed below, \Rightarrow
 - NBLLN and BRN are two of the four great under-extractions ... \Rightarrow
 - The two statistical ways that we infer too little from our environment. \Rightarrow
- But, in fact, depending on clumping issues, they are a killer combination. \Rightarrow
 - Suppose you are in an info-rich environment with infinite i.i.d. signals arriving sequentially. \Rightarrow
 - If suffered from NBLLN but not BRN, then learn when get signals one-by-one \Rightarrow
 - If suffered from BRN but not NBLLN, then learn when get signals all together. \Rightarrow
 - But ... \Rightarrow

The Fundamental Theorem of (Non-)Learning \Rightarrow

- If a person suffers from both NBLLN and BRN, then there does not exist a presentation of signals such that long-run beliefs converge to 

(Some) Problems with the simple BRN formula:⇒

- Similar problems other biases

For all the pretense to do PEEMish modification of Bayes:⇒

- Much left out of formulas that matter for updating.⇒
- Hard to mechanically apply.⇒

E.g., no formal difference “no information” vs “useless information.” ⇒

- Possible incremental improvement: if r.v. operates as signal, BRN even realization useless⇒

Beliefs depend on hypotheses focused on.⇒

- These framing effects are important and true⇒
- But make application of BRN sensitive to these framings.⇒
- And this is why BRN can lead to conjunction violations.

Another “problem” for BRN is an incompleteness \Rightarrow

- **Complete models for economics must say DM’s beliefs about future updating.** \Rightarrow
 - Evidence (on most biases) is retrospective \Rightarrow
 - Only asking what people think after seeing evidence. \Rightarrow
 - But (e.g., search) must know prospective. \Rightarrow
- In context of NBLLN, Benjamin, Rabin, and Raymond (2013): \Rightarrow
 - framework to think about “retrospective” vs “prospective” \Rightarrow
 - Needn’t be consistent \Rightarrow
 - E.g., Barney prospective separating, retrospective pooling. \Rightarrow
- Our best guess: \Rightarrow
 - Saki thinks “prospectively” that she’ll pay attention to base rates. \Rightarrow
 - Freddy (LSN) has consistent pro- and retro-spective.

Role of processing inconsistency \Rightarrow

- BRR show Barney may end up purchasing signals forever, \Rightarrow
 - with unbounded welfare loss! \Rightarrow
 - Prospective “acceptive” \implies Barney thinks new info can substantially change his beliefs. \Rightarrow But
 - retrospective pooling \implies impact of additional signal on beliefs small \Rightarrow
 - Inference driven increasingly by proportion of a signals, so marginal impact of a signal approaches zero. \Rightarrow
 - Barney always believes an additional signal will have the same impact. \Rightarrow

Questions? \Rightarrow

- And speaking of questions ... question-dependence of beliefs

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Support Theory and Bin Effects_⇒

- (Close cousin to stuff probably very familiar to you)_⇒
- Per Tversky and Koehler (1994) and other models, small probability events exaggerated, and combining events reduces total weight._⇒
- Suppose elicit beliefs p on exhaustive/exclusive 3 events $\{A, B, C\}$,_⇒
 - $p(A) + p(B) + p(C) = 1$._⇒
- And elicit q on 2 events $\{A \cup B, C\}$,_⇒
 - $q(A \cup B) + q(C) = 1$._⇒
- Then will find: $q(A \cup B) < p(A) + p(B)$ and $q(C) > p(C)$._⇒
 - **Important corollary: tendency to overestimate small probabilities**

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Cognitive Biases

Much literature. \Rightarrow BMR experiment (See Dan Benjamin or Don More website, not mine): \Rightarrow

- Massive bin effects. \Rightarrow
- And is some exaggeration of tail events in samples ... because we're asking low-probability events? \Rightarrow
- E.g., 400 participants all asked 12 questions ... \Rightarrow
 - 11 bins on 10 flips of coin \Rightarrow
 - And each of 11 questions "What is probability exactly N are heads?" \Rightarrow
 - Total of 1st forced to be 100%, but total of 11 other questions is 200% \Rightarrow
 - **ALL** N's are exaggerated. \Rightarrow
- Smoking-gun approach: \Rightarrow **Within partition:** \Rightarrow
 - when right answer $\pi(A) \leq \pi(B)$, subjects believe $p(A) > p(B)$ \Rightarrow
 - then bias towards A. \Rightarrow
 - (Working on more general exploration of method). \Rightarrow
- We show NLLN is real



Implications of Bin Effects? \Rightarrow

- Is it just an elicitation confound? \Rightarrow
 - To ID nature of bias, we “control” for it. \Rightarrow
 - But not clear that there is any such thing as “true” preference, independent from bin effects. \Rightarrow
 - And, presumably, also a real thing economically. \Rightarrow
- If people act as if the total probability of cancer is higher when broken down by different types, then ... \Rightarrow
 - Means structure of doctor communication, insurance, etc., matter.

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Now ... let us revisit general categories of biases in belief updating \Rightarrow

- And general features of updating that might differ from Bayesian \Rightarrow

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Cognitive Biases

If considering updating on binary state of world with binary signal, then: \Rightarrow

- Bayesian: $\ln\left(\frac{p_{t+1}}{1-p_{t+1}}\right) = \ln\left(\frac{p_t}{1-p_t}\right) + \ln\left(\frac{\sigma_t}{1-\sigma_t}\right)$, where
- p_t are priors, σ_t is new information, and p_{t+1} are posteriors. \Rightarrow

Can think of various ways might depart from (“Tommy”) Bayes:

- Non-Bayesian: $\ln\left(\frac{p_{t+1}}{1-p_{t+1}}\right) = \alpha \ln\left(\frac{p_t}{1-p_t}\right) + \beta \ln\left(\frac{\sigma_t}{1-\sigma_t}\right) \Rightarrow$
- Depart from $\alpha = \beta = 1$ is non-Bayesian.

Could overweight or underweight each of two components. \Rightarrow

- **Depending on the nature of the signals** \Rightarrow
- Unlikely that $\alpha < 0$ or $\beta < 0$. \Rightarrow
- Note: this is just retrospective, not prospective.



Can characterize known biases within framework: \Rightarrow

- Non-Bayesian: $\ln\left(\frac{p_{t+1}}{1-p_{t+1}}\right) = \alpha \ln\left(\frac{p_t}{1-p_t}\right) + \beta \ln\left(\frac{\sigma_t}{1-\sigma_t}\right) \Rightarrow$
- $\alpha < 1$ is base-rate neglect (Saki)
- $\beta > 1$ when σ_t is small sample is LSN (Freddy)
- $\beta < 1$ when σ_t is large sample is NBLLN (Barney)
- Confirmation bias: And Joely is ... \Rightarrow sort of $\alpha > 1 \Rightarrow$
 - when σ_t is ambiguous? \Rightarrow
 - when p_t are theory-based? \Rightarrow
- Of course: \Rightarrow
 - We are all Saki, Freddy, Barney, and Joely... \Rightarrow
 - See Dan Benjamin for meta-analysis on Saki, Freddy, and Barney

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What are implications of Bayes that matter? \Rightarrow

- Which ones violated by observed biases? \Rightarrow
- (“Matter” means we should think about economic implications.) \Rightarrow

In Bayesian , some fundamental features we’ve already seen violated: \Rightarrow

- Frame independence \Rightarrow
- Conjunction ($p(A \cap B) \leq p(A)$) \Rightarrow
- Prospective beliefs vs. retrospective same \Rightarrow
 - Self-based law of iterated expectations \Rightarrow
 - Observer-based law of iterated expectations \Rightarrow
- Add correct priors and \Rightarrow
 - Beliefs should generally be unbiased.



Cognitive Biases

More properties of Bayesian updating: \Rightarrow

- Sloppily stated \Rightarrow Mostly mean binary states. \Rightarrow

Retrospective beliefs: \Rightarrow

- Agent is Bayesian. \Rightarrow (proof omitted) \Rightarrow
- Fixing priors, then higher signal \rightarrow higher posterior. \Rightarrow
- Fixing signal, then higher priors \rightarrow higher posterior. \Rightarrow
 - Aside, some claims by neuro people that the brain is “Bayesian” ... \Rightarrow
 - really only mean the weakest possible features — they only ask the two above. \Rightarrow
- Tiny bit stronger: \Rightarrow
 - Fixing priors, then positive signal \rightarrow posterior $>$ priors. \Rightarrow
 - We’ve already seen that violated...



Prospective and retrospective beliefs?

- Prospective beliefs vs. retrospective same \Rightarrow
 - Self-based law of iterated expectations? \Rightarrow
 - Observer-based law of iterated expectations? \Rightarrow
- Prospective beliefs Bayesian? \Rightarrow

Something we won't talk much about, but also crucial: \Rightarrow

- Our beliefs about others' information processing? \Rightarrow
- And our beliefs about what they think our information processing is? \Rightarrow
 - And how we update our beliefs once we figure out that what their beliefs are? \Rightarrow

Questions?



Cognitive Biases

Long-run beliefs when getting i.i.d. signals? \Rightarrow

- Bayesian settles down on the truth. \Rightarrow

Others? \Rightarrow

- Settle down? \Rightarrow
- Settle down on truth? \Rightarrow
- Settle down on full confidence? \Rightarrow
- Priors matter? \Rightarrow
 - (For Bayesian? \Rightarrow They don't matter) \Rightarrow
- Path independence? Fixing truth, priors \rightarrow fully know limit ergodic distribution.

\rightarrow

Cognitive Biases

Let's review these long-run properties for errors discussed: \Rightarrow

- Saki? Barney? Joely? \Rightarrow

BRN NBLLN Confirm Tommy

settle?	yes
confident?	yes
always right?	yes \Rightarrow
priors matter?	no
path matters?	no
truth matters?	yes

- Thoughts?

\rightarrow

Cognitive Biases

Let's start with long-run beliefs: \Rightarrow

- Saki? \Rightarrow

	BRN	NBLLN	Confirm	Tommy
settle?	no			yes
confident?	no			yes
always right?	no			yes \Rightarrow
priors matter?	no			no
path matters?	no*			no
truth matters?	yes*			yes

- Thoughts?

\rightarrow

Cognitive Biases

Barney? \Rightarrow

	BRN	NBLLN	Confirm	Tommy
settle?	no			yes
confident?	no			yes
always right?	no			yes \Rightarrow
priors matter?	no			no
path matters?	no*			no
truth matters?	yes*			yes

- Thoughts?

\rightarrow

Cognitive Biases

Joely (haven't really studied her)?_⇒

	BRN	NBLLN	Confirm	Tommy
settle?	no	yes		yes
confident?	no	no		yes
always right?	no	no		yes _⇒
priors matter?	no	yes		no
path matters?	no*	no		no
truth matters?	yes*	yes		yes

- Thoughts?

↪

Cognitive Biases

And so: \Rightarrow

	BRN	NBLLN	Confirm	Tommy
settle?	no	yes	yes	yes
confident?	no	no	yes	yes
always right?	no	no	no	yes \Rightarrow
priors matter?	no	yes	yes	no
path matters?	no*	no	yes	no
truth matters?	yes*	yes	yes*	yes

- What does this say about learning models?

\Rightarrow

Cognitive Biases

- Now ... back to non-long-run belief patterns. \Leftarrow
- Mentioned an interest in “volatility” and confidence. \Leftarrow

Define two facets of each period \Leftarrow

- For binary events, look at changes in beliefs over time. \Leftarrow

Define (expected) belief **movement**: \Leftarrow

- $m_t \equiv E \left\{ \sum (p_{t+1} - p_t)^2 \right\} \Leftarrow$
 - This is how “volatile” your beliefs are. \Leftarrow

Define (expected) **uncertainty reduction**: \Leftarrow

- $r_t \equiv E \left\{ \sum [p_t(1 - p_t) - p_{t+1}(1 - p_{t+1})] \right\} \Leftarrow$
 - This is how confident you are becoming. \Leftarrow
 - (An alternative measure of entropy) \Leftarrow

\Leftarrow

Cognitive Biases

Why these exact measures? \Rightarrow

- What is the relationship between the two for Tommy? \Rightarrow
- **All Bayesian processes** beginning from correct priors: \Rightarrow
 - $m_t = r_t$ \Rightarrow
- Key intuition on rationality of updating: \Rightarrow
 - If beliefs changing, better be learning something: \Rightarrow
 - If a lot of movement, better be resolution of uncertainty. \Rightarrow
 - [Harder: if resolution of uncertainty, better see lots of movement]

Formal theorem simple, appears obscurely various places. \Rightarrow

- Augenblick & Rabin (2017) make big stink about. \Rightarrow
 - We show theory, lots evidence of excess movement \Rightarrow
- See Augenblick and Lazarus (2017) applying the principles... \Rightarrow
 - Show excess volatility in the stock market



Under different names, m and r studied aspects of statistical errors \Rightarrow

- Probably r is, with mistaken belief it is likely to be general. \Rightarrow
- And often it seems people conflate m and r \Rightarrow
- E.g., base-rate neglect on average \Rightarrow
 - seemingly over-reacts to information ... \Rightarrow
 - but is underconfident in her beliefs. \Rightarrow

\rightarrow

Most common tests of bias ... compare to the normative beliefs. \Rightarrow

- Augenblick and Rabin (2017) [Augenblick's website]:
- Even without knowing correct beliefs, we can compare m vs. r ... \Rightarrow
 - Martingale tests \Rightarrow
 - Except Bayesian beliefs are **bounded** Martingales \Rightarrow
 - Helps with statistically testing \Rightarrow
 - And m and r relationship to bias make $m - r$ good candidate for bias.
 \Rightarrow
- Can test for correct-prior Bayesian

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Cognitive Biases

- Reminder:

If considering updating on binary state of world with binary signal, then: \Rightarrow

- Bayesian: $\ln\left(\frac{p_{t+1}}{1-p_{t+1}}\right) = \ln\left(\frac{p_t}{1-p_t}\right) + \ln\left(\frac{\sigma_t}{1-\sigma_t}\right)$, where
- p_t are priors, σ_t is new information, and p_{t+1} are posteriors. \Rightarrow

Can think of various ways might depart from (“Tommy”) Bayes:

- Non-Bayesian: $\ln\left(\frac{p_{t+1}}{1-p_{t+1}}\right) = \alpha \ln\left(\frac{p_t}{1-p_t}\right) + \beta \ln\left(\frac{\sigma_t}{1-\sigma_t}\right) \Rightarrow$
- Depart from $\alpha = \beta = 1$ is non-Bayesian.

Could overweight or underweight each of two components. \Rightarrow

- **Depending on the nature of the signals** \Rightarrow
- Unlikely that $\alpha < 0$ or $\beta < 0$. \Rightarrow
- Note: this is just retrospective, not prospective.

Cognitive Biases

Movement vs. uncertainty resolution: \Rightarrow

- Biases vs. Tommy (m^T, r^T)
- Claims not quite true. \Rightarrow

	common manifestation: (priors, signal)	belief movement	uncertainty resolution	
Tommy	$\alpha = 1, \beta = 1$	$m^T = m^T$	$r^T = r^T$	\Rightarrow
Saki	$\alpha < 1, \beta = 1$	$m^S > m^T$	$r^S < r^T$	
Freddy	$\alpha = 1, \beta > 1$	$m^F > m^T$	$r^F > r^T$	
Barney	$\alpha = 1, \beta < 1$	$m^B < m^T$	$r^B < r^T$	
Joely	$\alpha > 1, \beta = 1$	$m^J < m^T$	$r^J > r^T$	

- Thoughts, surprises? \Rightarrow
 - Reminder ... not quite true ...



Cognitive Biases

Movement vs. uncertainty resolution \Leftrightarrow

- Even if don't know *correct* Tommy (m^T, r^T), we know to expect $m = r$
- More claims not quite true (remember, always in expectation) \Leftrightarrow

	common manifestation: (priors,signal)	belief movement	uncertainty resolution	$m - r$	
Tommy	$\alpha = 1, \beta = 1$	$m^T = m^T$	$r^T = r^T$	0	
Saki	$\alpha < 1, \beta = 1$	$m^S > m^T$	$r^S < r^T$	+	
Freddy	$\alpha = 1, \beta > 1$	$m^F > m^T$	$r^F > r^T$	+	\Leftrightarrow
Barney	$\alpha = 1, \beta < 1$	$m^B < m^T$	$r^B < r^T$	-	
Joely	$\alpha > 1, \beta = 1$	$m^J < m^T$	$r^J > r^T$	-	
Noisy, observe $\pm \epsilon$				+	
Overstrong priors				+	
Overweak priors				-	

So:

Beliefs?

Let's get out there and study beliefs!

- Do they know how much debt they'll be in 3 years from now?
 - How much debt others will be in?
- Beliefs about whether their financial advisor has a fiduciary responsibility?
- What do they think the typical economist believes about investing in own company's stock?
- What do they believe random climate scientist at Universities of Oklahoma, Utah, and Alabama believes?
- What was their own rate of return on investment last year? What was Vanguard's?
- *Etc.*



Beliefs?

Suppose that a psychologist proposes “9-11 Bias”:

- Because of the salience of that number in the, well, post-9-11 world, claimed that people think anything more likely if evokes 9-11. \Rightarrow
 - And shows that when ask people the likelihood that you get exactly 911 heads out of 1,000 — or 911 tails out of 1,000 — that people overstate! Instead of ϵ , they say a few percentage points. \Rightarrow
- Or, for that matter, what if researchers claim people exaggerate likelihood of terrorist attack ... \Rightarrow
 - By asking people likelihood of terrorist attack \Rightarrow
- Reminder: depending on model of world — and if asking whether talking about terrorism makes people think terrorism is more likely — then these are the right questions.
 - But misleading to think people especially attuned to it. \Rightarrow
 - By same token, you'll exaggerate likelihood of big flood if ask about it, mass suicide if ask about it, SARS outbreak, undefeated season for Cal Tech basketball, likelihood Mt. Everest is exactly 30,000 feet, likelihood it is exactly 5,000 feet, likelihood I'll win the World Cup (?) \Rightarrow ... \Rightarrow

Beliefs?

Beliefs about happiness! \Rightarrow

- Interested in the topic ... \Rightarrow
- What do people think is source of happiness? Who do they think are happyh?
- I asked you in 4 different orders. \Rightarrow
 - Which is the right one? \Rightarrow
- P.s.: Any volunteers to calculate m and r ?

\curvearrowright

We should gather direct evidence on beliefs across domains. ⇐⇒

- Is happening some ⇐⇒
- Should happen more ⇐⇒
 - **Will** happen more ⇐⇒

↷

- **For identifying biases, we need histograms.**⇒
 - So please help⇒
- Help us understand how widespread lab errors are in world⇒
 - Evidence is on coins & urns, but want evidence on realer things.⇒
- People's beliefs about how others would interpret given information.⇒
 - Directly gets at issues of interpersonal thinking.⇒
 - But also indirectly at nature of biases, especially confirmatory.⇒
- **Prospective beliefs**⇒
 - Huge: beliefs about future updating⇒
 - Virtually unmentioned in the psychology⇒
 - We've been making models without any evidence

Models of Inference and Learning from Others_⇒

Others' behavior might reveal information to us._⇒

- How good gleaning information from others?_⇒
- What systematic errors?_⇒
- What effects of these errors?_⇒

Earlier:_⇒

- Failures of statistical reasoning._⇒

Now: _⇒

- Errors in inference from volitional agents, per se



Inference and Learning from Others

What mean "volitional agents per se"? \Rightarrow

- In fact, Behavioral GT weird history...
 - Mistakes in game any different than elsewhere? \Rightarrow
 - When see sample of independent "people flips" may make same errors as when see sample of independent coin flips. \Rightarrow

Volitional agents: we can (or should!) use our understanding of their motives as additional source of information. \Rightarrow

- What mistakes we make in that form of reasoning? \Rightarrow

\curvearrowright

Inference and Learning from Others

Will tell you about two facts about your interactions with others: \Rightarrow

- 1 Their behavior often contains information relevant to you which you do not glean in other ways. \Rightarrow
- 2 Information in two or more people, when they themselves interact, will be correlated by the logic of their interaction. \Rightarrow
 - Because they themselves are gleaning information. \Rightarrow

Eyster and Rabin propose: \Rightarrow

- We tend to under-attend to info content. \Rightarrow
- And tend to under-attend to correlation. \Rightarrow

Lots of things in the world contain information! \Rightarrow

- We often neglect them \Rightarrow
- Often see it when not there!



Inference and Learning from Others

Cursed Thinking: \Rightarrow

- People under-infer information from others' behavior. \Rightarrow
 - Winner's curse in common-values auctions: \Rightarrow
 - Lemons and **financial markets** \Rightarrow
 - No no-trade results—naturally and directly and disciplinedly \Rightarrow

Naive Inference: \Rightarrow

- Insofar as *do* attend to information in others' behavior, tend to take at face value. \Rightarrow
 - People don't attend to *redundancy* in social beliefs. \Rightarrow
- In observational learning: \Rightarrow
 - Rationality predicts some imitation ... \Rightarrow
 - but typically predicts anti-imitation too \Rightarrow
 - and *precludes* extensive imitation without anti-imitation \Rightarrow
 - naive inference, redundancy neglect \implies ubiquitous imitation \Rightarrow
 - overconfidently wrong social beliefs

Topics \Rightarrow

① Cursedness \Rightarrow

- ① Motivation and evidence \Rightarrow
- ② Implications in asset markets \Rightarrow
- ③ Compared to (overconfident) “agreeing to disagree” \Rightarrow

② Rational and Naive Observational Learning \Rightarrow

- ① **Behavioral** implications of rational observational learning \Rightarrow
- ② Informational, societal consequences of naive herding \Rightarrow

↗

“Cursedness” \Rightarrow

- Eyster and Rabin (2005) “cursed equilibrium” \Rightarrow
 - From “Winner’s Curse” in Auctions: \Rightarrow
 - Bid your estimate and win \rightarrow others more negative information. \Rightarrow
 - People don’t fully think through the informational content in others’ behavior. \Rightarrow
- **Interpretation: Limited Rationality**, \Rightarrow
 - **As if** think others’ behavior independent of their information? \Rightarrow
 - Because eqbm, would be very strange. \Rightarrow
 - **Not** interpreted thusly. \Rightarrow
 - **Rather:** inattentiveness to the fact that they are doing so. \Rightarrow
- Strong (& tenuous) feature: \Rightarrow
 - correctly predict others’ average *behavior*. \Rightarrow
- **Portable and pinned-down** model of “information neglect” \Rightarrow
 - Formalism misses lots \Rightarrow
 - (including lots of psych that is part of motivation)

Winner's curse in trade/lemons: Example \Rightarrow

- Firm has book value $v \sim U[0, 1]$. \Rightarrow
- Firm knows book value, Raider does not. \Rightarrow
- Firm values at v and Raider at $\frac{3}{2}v$. \Rightarrow
- Raider makes TIOLI offer b to Firm. \Rightarrow
- PBE: Firm sells iff $v < b \implies$ average value $\frac{3}{2} \left(\frac{1}{2}b\right) = \frac{3}{4}b < b$ \Rightarrow
- Optimal (lemon-flavoured) bid: $b^* = 0$ \Rightarrow
- Experimental Evidence: Bazerman and Samuelson (1985):
 - 59% of subjects bid in $[0.5, 0.75]$, and 92% bid more than 0. \Rightarrow
 - Subjects lose money on average.

↪

Cursed equilibrium: \Rightarrow

- Firm accepts iff $v < b$.
- Raider understands higher $b \implies$ more likely accepted: \Rightarrow predicts true $\text{prob}(b \text{ accepted}) = b$. \Rightarrow
 - Doesn't attend to lemons problem when bidding? \Rightarrow
- As if Raider thinks sales independent of v . \Rightarrow
- Raider solves $\max_b b(\frac{3}{2}E[v] - b) = \frac{3}{4}b - b^2$ and $b^* = \frac{3}{8} = .38$. \Rightarrow
- (Note: even most extreme form of our model too low.)

\curvearrowright

Eyster, Rabin, and Vayanos (2013): \Rightarrow

- Adapt CE to market model. \Rightarrow
- Cursed traders don't think through information in market prices. \Rightarrow
 - Don't ask: why is that person trading with me? \Rightarrow
 - Retro! ... pre-Grossman REE revolution. \Rightarrow
 - Old school not 100% wrong \Rightarrow
- Simple, one-period asset market (with returns realized after trade) \Rightarrow
- Variously all cursed vs. some rational, some cursed.

\curvearrowright

Inference and Learning from Others

CEE vs. “REE”: \Rightarrow

- Many specific predictions (e.g., “momentum”) \Rightarrow
- In REE: \Rightarrow
 - Traders might trade for liquidity, etc. \Rightarrow
 - But no “speculative trade” \Rightarrow
 - Not plausible volume. \Rightarrow
- Biggest and simplest implication of CEE: \Rightarrow
 - **High volume of trade.** \Rightarrow
- Rational traders: more private info \implies less trade. \Rightarrow
 - Rationals become more suspicious of others’ motivation to trade \Rightarrow
- With cursedness: more private info can imply *more* trade. \Rightarrow
 - Increasing difference in opinion, w/o understanding adverse selection.



Alternative approaches? \Rightarrow

- Overconfidence \Rightarrow
 - People think own signals are better than they are. \Rightarrow
 - (And maybe that others' signals worse)
- **Agreeing to disagree, non-common priors** \Rightarrow
 - ATD directly, or to “close” overconfidence models. \Rightarrow
 - Traders aware of disagreements. \Rightarrow
 - Seek bets based on this! \Rightarrow
 - (This explanation ... massive degrees of freedom) \Rightarrow
 - And often not plausible: \Rightarrow
- ERV show experimental evidence \Rightarrow
 - Where overconfidence incoherent \Rightarrow
 - Priors don't matter

Inference and Learning from Others

Return to Bazerman and Samuleson: \Rightarrow

- Recall: uninformed Raider over-bids on perfectly informed Firm. \Rightarrow
- Raider overbidding because of “overconfidence”? \Rightarrow
 - Knows nothing, and surely knows she knows nothing. \Rightarrow
- Raider agreeing to disagree with Firm’s beliefs on v ? \Rightarrow
 - Surely believes Firm is exactly right. \Rightarrow
 - Rather: she is not fully thinking through the lemons problem. \Rightarrow

Repeated in much of other evidence: \Rightarrow

- People behave in ways that look like market trading due to overconfidence and ATD. \Rightarrow
- But in settings where neither could be going on. \Rightarrow

Doesn’t prove OC/ATD isn’t going on in markets! \Rightarrow

- But clarifies alternative, and suggests worry.



Inference and Learning from Others

ATD driving force of small investors trading against the market? \Rightarrow

- **Do they think they are outsmarting experts and insiders?** \Rightarrow
- **Or not attending that they are trading against them?** \Rightarrow
- May think they are aligned with “smart money” against fools? \Rightarrow

ERV shows that, even if true, overconfidence/ATD unlikely to explain high volume in large, “high-information” markets. \Rightarrow Roughly: \Rightarrow

- If a trader believes \Rightarrow
 - lots of information besides her own insights, \Rightarrow
 - then just like REE logic \implies little speculative trade in large markets. \Rightarrow
 - Degree overconfidence needed w/o cursedness?



Inference and Learning from Others

“Cursedness” is a necessary, and huge, error **on top of everything else.**⇒

- Generalization: failure to use the logic of a market exchange to bound your probabilistic beliefs.⇒

General lessons in thinking about errors.⇒

- Blah blah blah ... complexity ... ⇒
- Suppose Rob (carrying a calculator) walks up to Roy and offers:⇒
 - “You give me \$100 if the 12th + 13th digit of π^2 are 93. I give you \$100 if anything else. Decide immediately without electronics.” ⇒
 - Suppose Roy says “Yes” — and (bad luck, those digits are 93!) pays \$100.⇒
 - What mistake is Roy making?⇒
- With limited learning, limited attention, bounded rationality ⇒
 - Even when definitely, provably necessary role ⇒
 - It is not the epistemologically, scientifically relevant “explanation”
- **Celerier & Vallee (2015)** paper ... shocking stuff.



Generalization of “cursedness” \Rightarrow

- “Inversion neglect”: Even when paying attention to variables, do people have *any* consistent “structural” beliefs whatsoever to explain their reaction? \Rightarrow
- Do people form a theory of their credit-card behavior, insurance that is consistent with companies trying to make money? \Rightarrow
- Form a theory medicine and illness consistent with the world they see? \Rightarrow

\rightarrow

Inference and Learning from Others

- Shrouded attributes ... \Rightarrow

What are people's beliefs about what they are buying? \Rightarrow

- We are using the term (legitimately) for two importantly different things:
 - Not knowing some feature that no intrinsic reason to know about \Rightarrow
 - And key Gabaix-Laibson insight that market incentives won't de-shroud \Rightarrow
 - But per se that insurance companies, credit-card companies, or any company intends to make money off of you **should** deshroud the shrouded attribute \Rightarrow
 - And you should know an ointment won't make you live forever or cure cancer ... \Rightarrow
 - Because you don't see immortals around you ... and you see people dying from cancer. \Rightarrow Why?

But modeling shrouded attributes of either type: \Rightarrow

- Whether or not people neglecting a logic they should know about ... \Rightarrow
- You must have a model about what they are thinking, not just what

Inference and Learning from Others

- A short history of Vitamin C_⇒ (For guinea pigs, fruit bats, and primates)_⇒
- Dawn of Humanity until Age of Oceanic Voyages (Exploration, Imperialism, Migration): People unknowingly avoided scurvy_⇒
 - and got colds whenever they got colds._⇒
- Age of Oceanic Voyages until discovery of citrus fruit: Huge numbers of people died at sea of scurvy
 - and got colds whenever they got colds._⇒
 - [Some folk wisdom and health officials ignored or forgotten]_⇒
 - [People paid too much attention to ship hygiene and to Argentinians]_⇒
- Dawn of Age of Citrus at Sea until end of Age of Oceanic Voyages: Huge numbers of people intentionally avoided scurvy
 - and got colds whenever they got colds._⇒
- End of Age of Oceanic Voyages until 1970: People inattentively avoided scurvy,
 - and got colds whenever they got colds._⇒

Inference and Learning from Others

- A short history fo Mercury \Rightarrow

Humankind discovers Mercury. It's totally cool. It's used for medicine. \Rightarrow

- Late 1400's: syphilis arrives in Europe and Europe-settled areas. \Rightarrow
- Mercury was used on massive scale as a treatment for syphilis \Rightarrow
- Syphilis and hats kill millions and drive millions crazy \Rightarrow
 - Heads get covered, armies shut down \Rightarrow
- Different types of errors ... \Rightarrow
 - not knowing about a poison (hats) \Rightarrow
 - Administering a poison intentionally \Rightarrow
- Early 20th century: Mercury combined with arsenic as treatment for syphilis \Rightarrow
- February 28, 1928: Fleming discovers penicillin (430 years after syphilis) \Rightarrow
- Syphilis (and much else) virtually shut down overnight. \Rightarrow



Inference and Learning from Others

- A short history of blood-letting ... \Rightarrow
- A short history of housing \Rightarrow
 - We all know the prices always go up \Rightarrow
 - We all stopped knowing that \Rightarrow
- A short history of tulips \Rightarrow
 - (But I like my “bubbles” to last for centuries) \Rightarrow
- A short history of investing retirement savings in individual stocks \Rightarrow
- A short history of whiplash \Rightarrow
 - Fast things invented. \Rightarrow
 - Rear-ending “invented” \Rightarrow
 - Thousands stayed in bed for millions of weeks over a decade \Rightarrow
 - Oh, wait, that doesn't help



Inferential Naivety in Observational Learning \Rightarrow

- People may extract information (not fully cursed) from others. \Rightarrow
- But take this information “at face value” \Rightarrow
- Portable and pinned down universal definition \Rightarrow
 - But now talk solely about observational learning.

↪

Rational-Herding Literature: \Rightarrow

- People infer from actions of those with similar tastes. \Rightarrow
- Rational imitation. \Rightarrow
- Herds may start & last on wrong choice. \Rightarrow
- All realize that others also imitating \Rightarrow
- Understand *inherent* redundancy in others' behavior \Rightarrow
- Don't imitate very much.

\rightarrow

Inference and Learning from Others

- We are skeptical people so reluctant to imitate. \Rightarrow
- **And we should care a lot about this:** \Rightarrow
 - Examples of theories that generate extensive imitation ... predict severe badnesses in societal beliefs. \Rightarrow
- Extensive imitation \Rightarrow not rational \Rightarrow
- Extensive imitation \Rightarrow social confirmation bias & false beliefs. \Rightarrow

Now: \Rightarrow

- **Behavioral implications** of full rationality in observational learning \Rightarrow
- Very different implications of inferential naivety.

\curvearrowright

Review of Canonical Rational-Herding Models \Rightarrow

Banerjee (1992), Bikhchandani, Hirshleifer and Welch (1992). \Rightarrow

- Sequentially move, all privately informed, observe actions and order but not info of those before. \Rightarrow
- No direct externalities. \Rightarrow
- Canonical example:

↪

Inference and Learning from Others

Canonical model: Either A or B is good—but not both (?)—and binary private i.i.d. signals. A when you believe $\omega = 1$, B when $\omega = 0$. α signal of $\omega = 1$, β of $\omega = 0$; \Rightarrow

<u>player</u>	<u>signal</u>	<u>action</u>
1	α	A
2	β	B
3	α	A
4	α	A
5	β	[A]
6	α	[A]
7	β	[A]
8	α	[A]
9	β	[A]
10	β	[A]



Efficiency facts of rational herding models: \Rightarrow

- Observing others always helps in expected terms. \Rightarrow
- High likelihood wrong herds only if those herds are unconfident. \Rightarrow
- Rational-herding literature is about failure to aggregate information, not of society (frequently) thinking it knows things it doesn't. \Rightarrow
 - (Debated in literature: is even non-aggregation really likely?)

↪

Inference and Learning from Others

We claim: \Rightarrow

- Canonical example, connotation of literature misleading. \Rightarrow
- Limits to imitation far bigger punchline than the imitation itself. \Rightarrow
- We think the non-imitation is unrealistic. \Rightarrow
- And (later show) it matters. \Rightarrow

Modification of the standard two-restaurant model of social learning. \Rightarrow

- Two restaurants in town, \Rightarrow
 - A and B , $p(A \text{ good}, B \text{ bad}) = p(B \text{ good}, A \text{ bad}) = .5$. \Rightarrow
 - Two states: $\omega_A \rightarrow A$ is good, $\omega_B \rightarrow B$ is good. \Rightarrow
 - Binary-state model universal. \Rightarrow ... and weird.

\curvearrowright

Inference and Learning from Others

- Each of ∞ diners receives private signals $\in \{\alpha, \beta, \emptyset\}$
- The signals are *i.i.d.* conditional on the state,
 - α supports ω_A ,
 - β supports ω_B ,
 - \emptyset uninformative.
- For each Player k ,
 - $\Pr[s_k = \alpha | \omega_A] = \Pr[s_k = \beta | \omega_B] = .7(1 - \eta)$ and
 - $\Pr[\emptyset | \omega_A] = \Pr[\emptyset | \omega_B] = \eta$.
 - $\eta = 0$, canonical binary-signal information structure.
 - When $\eta \rightarrow 1$, information is very rare.
 - (Lots results independent of η)

↷

Inference and Learning from Others

- Each Player k chooses among nine choices: \Rightarrow
 - dine in Restaurant A, dine in Restaurant B, or dine at home. \Rightarrow
 - Goes to a restaurant if she thinks there is more than 60% chance it is good, and stays at home if that is not true at either restaurant. \Rightarrow
- Depending on confidence in restaurant's quality, may go alone, or take one, two, or three of her relatives. \Rightarrow
- Superscripts for the number of people she takes: \Rightarrow

$p(\omega_A)$ $[0,10), [10,20), [20,30), [30,40)$ $[40,60]$ $(60,70], (70,80], (80,90], (90,100]$

Choice B^{+++}, B^{++}, B^+, B H A, A^+, A^{++}, A^{+++}

\Downarrow

Three people choose restaurants each period, \Rightarrow

- Signal conditionally i.i.d. given state \Rightarrow
- Each doing so after observing her own signal, \Rightarrow and the full actions (three locations, and party size), in order, \Rightarrow taken in all previous periods.

This example is clearly very contrived. \Rightarrow But scout's honor ... \Rightarrow

- General punchlines not based on specifics.

↪

Inference and Learning from Others

What predictions does full rationality make? \Rightarrow

- \emptyset signal, observes nothing but $H \rightarrow$ stay home. \Rightarrow
- α or β signal, observes nothing but $H \rightarrow$ go to restaurant. \Rightarrow
- (alone, because beliefs exactly .7 \rightarrow alone). \Rightarrow

Suppose in period 2 observe that exactly one person has gone to Restaurant A in period 1. \Rightarrow

- What do as a function of your signal? \Rightarrow
- You will realize that the three signals in period 1 were $\{\alpha, \emptyset, \emptyset\}$. \Rightarrow
 - $\beta \rightarrow H$. \Rightarrow
 - $\emptyset \rightarrow A$. \Rightarrow
 - $\alpha \rightarrow A^{++}$

\rightarrow

Inference and Learning from Others

If observe: \Rightarrow

actions

Period 1: $\{A, H, H\} \Rightarrow$

Period 2: $\{A, A, A\}$

What do (as function of signal)?

\Downarrow

Inference and Learning from Others

	actions	response	signals
Period 1:	$\{A, H, H\}$		$\{\alpha, \emptyset, \emptyset\}$
Period 2:	$\{A, A, A\}$		$\{\emptyset, \emptyset, \emptyset\}$
Period 3:		$\beta \rightarrow H, \emptyset \rightarrow A, \alpha \rightarrow A^{++}$	

- Key logic: guys in period 2 did *not* get any additional information. \Rightarrow
 - (If did, would not have gone alone.) \Rightarrow
 - Period 3: rationally realize no new information in Period-2 followers.

\rightarrow

Inference and Learning from Others

If observe: \Rightarrow

actions

Period 1: $\{A, H, H\}$
Period 2: $\{A, A, A\} \Rightarrow$
Period 3: $\{A, A, A\}$
Period 4: $\{A, A, A\}$
Period 5: $\{A, A, A\}$

What do as (as function of signal)?

\rightarrow

Inference and Learning from Others

	actions	response	signals
Period 1:	$\{A, H, H\}$		$\{\alpha, \emptyset, \emptyset\}$
Period 2:	$\{A, A, A\}$		$\{\emptyset, \emptyset, \emptyset\}$
Period 3:	$\{A, A, A\}$		$\{\emptyset, \emptyset, \emptyset\}$
Period 4:	$\{A, A, A\}$		$\{\emptyset, \emptyset, \emptyset\}$
Period 5:	$\{A, A, A\}$		$\{\emptyset, \emptyset, \emptyset\}$

Period 6: $\beta \rightarrow H, \emptyset \rightarrow A, \alpha \rightarrow A^{++}$

- Understanding redundancy information in actions: hard. \Rightarrow
- But it matters a lot.

\curvearrowright

Inference and Learning from Others

If observe: \Rightarrow

actions

Period 1: $\{A, H, H\} \Rightarrow$

Period 2: $\{A, A, H\}$

What do (as function of signal)?

\rightarrow

Inference and Learning from Others

Herding without sufficiently increased enthusiasm is a bad sign:

	actions	response	signals
Period 1:	$\{A, H, H\}$		$\{\alpha, \emptyset, \emptyset\}$
Period 2:	$\{A, A, H\}$		$\{\emptyset, \emptyset, \beta\}$

Period 3: $\beta \rightarrow B, \emptyset \rightarrow H, \alpha \rightarrow A$
 \Rightarrow

3 A, 3 H $\rightarrow \omega_A, \omega_B$ equally likely!
 \Rightarrow

- Do we get that?

\rightarrow

Inference and Learning from Others

If observe: \Rightarrow

actions

Period 1: $\{A, H, H\} \Rightarrow$

Period 2: $\{A, H, H\}$

What do (as function of signal)?

$\varphi \rightarrow$

Inference and Learning from Others

	actions	response	signals
Period 1:	$\{A, H, H\}$		$\{\alpha, \emptyset, \emptyset\}$
Period 2:	$\{A, H, H\}$		$\{\emptyset, \beta, \beta\}$
Period 3:		$\beta \rightarrow B^{++}, \emptyset \rightarrow B, \alpha \rightarrow H$	

You shouldn't go to A even if get α !

\curvearrowright

Inference and Learning from Others

If observe: \Rightarrow

actions

Period 1: $\{A, H, H\}$ \Leftarrow

Period 2: $\{H, H, H\}$

What do (as function of signal)?

\Downarrow

Inference and Learning from Others

	actions	response	signals
Period 1:	$\{A, H, H\}$		$\{\alpha, \emptyset, \emptyset\}$
Period 2:	$\{H, H, H\}$		$\{ \beta, \beta, \beta \}$
Period 3:		$\beta \rightarrow B^{+++}, \emptyset \rightarrow B^{++}, \alpha \rightarrow B$	

Go to B no matter what!

$\varphi \rightarrow$

Inference and Learning from Others

If observe: \Rightarrow

actions

Period 1: $\{A, H, H\} \Rightarrow$

Period 2: $\{A^{++}, A, A\}$

Period 3: $\{A^{++}, A, A\}$

What do (as function of signal)?

$\varphi \rightarrow$

Inference and Learning from Others

	actions	response	signals
Period 1:	$\{A, H, H\}$		$\{\alpha, \emptyset, \emptyset\}$
Period 2:	$\{A^{++}, A, A\}$		$\Rightarrow \{\alpha, \emptyset, \emptyset\}$
Period 3:	$\{A^{++}, A, A\}$		$\{\emptyset, \beta, \beta\}$
Period 4:		$\beta \rightarrow B, \emptyset \rightarrow H, \alpha \rightarrow A$	

\curvearrowright

Inference and Learning from Others

If observe: \Rightarrow

actions

Period 1: $\{A, H, H\}$
Period 2: $\{A^{++}, A, A\}$ \Rightarrow
Period 3: $\{A^{++}, A^{++}, A\}$
Period 4: $\{A^{++}, A^{++}, A\}$
Period 5: $\{A^{++}, A^{++}, A^{++}\}$

What do (as function of signal)?

\mapsto

Inference and Learning from Others

	actions	response	signals
Period 1:	$\{A, H, H\}$		$\{\alpha, \emptyset, \emptyset\}$
Period 2:	$\{A^{++}, A, A\}$		$\{\alpha, \emptyset, \emptyset\}$
Period 3:	$\{A^{++}, A^{++}, A\}$		$\{\emptyset, \emptyset, \beta\}$
Period 4:	$\{A^{++}, A^{++}, A\}$		$\{\alpha, \alpha, \emptyset\}$
Period 5:	$\{A^{++}, A^{++}, A^{++}\}$		$\{\beta, \beta, \beta\}$
Period 6:		$\beta \rightarrow H, \emptyset \rightarrow A, \alpha \rightarrow A^{++}$	

Will a β signal help stop the herd?

↷

Inference and Learning from Others

If observe: \Rightarrow

actions

Period 1: $\{A, A, A\}$ \Rightarrow

Period 2: $\{A^{++}, A^{++}, A^{++}\}$

What do (as function of signal)?

\curvearrowright

Inference and Learning from Others

	actions	response	signals
Period 1:	$\{A, A, A\}$		$\{\alpha, \alpha, \alpha\}$
Period 2:	$\{A^{++}, A^{++}, A^{++}\}$		$\{\beta, \beta, \beta\}$
Period 3:		$\beta \rightarrow B, \emptyset \rightarrow H, \alpha \rightarrow A$	

- Enough. \Rightarrow
- Things far more complicated if \Rightarrow don't observe order, \Rightarrow don't observe all, or \Rightarrow heterogenous preferences \Rightarrow
 - But nothing makes the severe limits to imitation go away \Rightarrow
- **Others' beliefs massively correlated** \Rightarrow
 - \Rightarrow **musn't imitate too much.** \Rightarrow

\curvearrowright

Inference and Learning from Others

Same setting (same signals, players per period, etc.) but: \Rightarrow

- Cannot observe order of play. \Rightarrow
- **Signals rare** \Rightarrow
- In period 3, if see \Rightarrow
 - If see $\{H, H, H, H, H, H\}$, then believe $\Rightarrow .5$ \Rightarrow
 - If see $\{A, H, H, H, H, H\}$, then believe $\Rightarrow .7$ \Rightarrow
 - If see $\{A, A, H, H, H, H\}$, then believe $\Rightarrow .84$ \Rightarrow
 - If see $\{A, A, A, H, H, H\}$, then believe $\Rightarrow .5$ \Rightarrow
 - If see $\{A, A, A, A, H, H\}$, then believe $\Rightarrow .7$ \Rightarrow
 - If see $\{A, A, A, A, A, H\}$, then believe $\Rightarrow .3$ \Rightarrow

One old and one new example:

\rightarrow

Inference and Learning from Others

	actions	response	signals
Period 1:	$\{A, H, H\}$		$\{\alpha, \emptyset, \emptyset\}$
Period 2:	$\{H, H, H\}$		$\{\beta, \beta, \beta\}$
Period 3:		$\beta \rightarrow B^{+++}, \emptyset \rightarrow B^{++}, \alpha \rightarrow B$	

	actions	response	signals
Period 1:	$\{B, H, H\}$		$\{\beta, \emptyset, \emptyset\}$
Period 2:	$\{H, H, H\}$		$\{\alpha, \alpha, \alpha\}$
Period 3:		$\beta \rightarrow A, \emptyset \rightarrow A^{++}, \alpha \rightarrow A^{+++}$	

Anti-imitation!

Inference and Learning from Others

Lesson? \Rightarrow

- Choosing a restaurant may be hard! \Rightarrow

More interesting lesson: \Rightarrow

- Most natural learning environments... \Rightarrow
 - Great deal redundancy. \Rightarrow
- Theorem 1 \Rightarrow ('False but not misleading'): \Rightarrow
 - Almost all non-single-file environments, rationality implies 'anti-imitation'. \Rightarrow
 - Intuition: if two recent guys both imitating earlier guy but not each other—imitate both, \Rightarrow
 - **but subtract off source of correlation... earlier guy.** \Rightarrow
- Theorem 2: \Rightarrow
 - What few environments that don't demand 'anti-imitation' \Rightarrow (e.g., single-file), \Rightarrow imitate all you see as if seeing only one person.



Harder to see: \Rightarrow

- Rational observational learning in this case: \Rightarrow
 - Eventually will herd on $\{B^{+++}\}$ or $\{A^{+++}\}$. \Rightarrow
 - More than 95% of time \rightarrow right restaurant. \Rightarrow
 - Intuition: any lesser certainty, contrary signal will moderate behavior. \Rightarrow
- **When signals rare:** \Rightarrow
 - Roughly 30% of time herd starts in wrong direction, \Rightarrow
 - stays wrong $<$ 5% time. \Rightarrow
 - \rightarrow $>$ 25% of time: herd in wrong direction followed by reversal... \Rightarrow
 - somebody observing at least 50 people going to one restaurant and none to other decides stay home based on opposite signal.

\rightarrow

Inference and Learning from Others

Rationality does not just demand great care. \Rightarrow

- demands something people get wrong in systematic direction. \Rightarrow

What happens if instead people neglect redundancy? \Rightarrow

- ER (2010) particular extreme form. \Rightarrow
- “Britney” behaves as if all she is observing are independent. \Rightarrow

When signals are rare, \Rightarrow

- Tommy: \Rightarrow
 - less than 5% chance society converges to wrong restaurant, \Rightarrow
 - Everybody less than 96% sure they have right one. \Rightarrow
- Britney: \Rightarrow
 - $\sim 30\%$ chance convergence to wrong restaurant \Rightarrow
 - Everybody around $\sim 100\%$ sure they are right.



Inference and Learning from Others

Eyster-Rabin (2014) ... \Rightarrow

- In many environments, tiny amounts of naive redundancy neglect can lead society astray. \Rightarrow
- In example: \Rightarrow For weak signals, **any** learning rule in which \Rightarrow
 - everybody (?) \Rightarrow who sees 50 or more go one restaurant and none go to other follows the herd no matter signal \Rightarrow
 - will lead to 30% chance of false herd.

Deconstructing Overconfidence

By the way:

- I think Odean/Malmendier overconfidence, people being overly confident about their skills \Rightarrow
 - has a subtle connection with over-precision, \Rightarrow **is** a thing. \Rightarrow
- And over-optimism may (or may not) also be a thing. \Rightarrow
- **But:** *general* consequences are unclear \Rightarrow
- I teach the Fundamental Theorem of Overoptimism and Fundamental Theorem of Overconfidence in Talents: \Rightarrow
 - No behavioral predictions. \Rightarrow
- Proof: \Rightarrow What happens if choosing between options A and B, when instead of accurate $U(A)$, $U(B)$, have distorted beliefs $V(A) > U(A)$ and $V(B) > U(B)$? \Rightarrow Exactly. \Rightarrow
 - Only *comparative* confidence and optimism matter

Deconstructing Overconfidence

- In most cases, good sense prevails. \Rightarrow More distortion for things involving more uncertainty? More skill? \Rightarrow
 - But I think even smart folk sometimes give such overconfidence as empty explanation. \Rightarrow
 - Lots of things explain mistaken wars. \Rightarrow
 - But overconfidence isn't one of them, unless specify leaders more overconfident about winning wars than about diplomatic ability, etc.