

# Behavioral Agents in General Equilibrium

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## Outline:

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5. The Curse of Education: Educating the consumer makes her unprofitable
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# 1 Protective markets

- Suppose you think that eating an apple will save your life.
- So you value an apple at \$10,000,000.
- Happily, your mistake won't cost you anything.
- The competitive market sells apples to everyone at the same price.

## 1.1 Exploitative markets

- Suppose you think that only *my* farm's apples will save your life.
- So you value my apples at \$10,000,000 (and I know this).
- Unhappily, your mistake will cost you.
- I am a monopoly provider of my farm's apples.

## 2 Dutch Books

Economists use Dutch Book — aka money pump — arguments to rule out some tastes and beliefs.

1. Suppose a consumer had non-classical preferences.
2. Then the consumer would participate in a series of self-impoverishing trades referred to as a Dutch Book or a money pump.

The third step in the argument comes in two different versions:

- 3a. Self-improverishing trades will bankrupt the consumer, implying that such consumers will not play an important role in the economy.
- 3b. Self-improverishing trades are empirically uncommon, so preferences that imply such trades must also be uncommon.

## 2.1 Present bias and Dutch Books

An example: A model of three days of consumption.

Consider an agent (“Naif”) with a quasi-hyperbolic discount function.

$$U_1 = \ln c_1 + (1/2)[\ln c_2 + \ln c_3]$$

$$U_2 = \ln c_2 + (1/2)[\ln c_3]$$

$$U_3 = \ln c_3.$$

Naif believes (incorrectly) that his preferences in period 2 are dynamically consistent with his preferences in period 1.

So Naif believes that  $U_2 = \ln c_2 + \ln c_3$ .

Self 1’s ideal consumption stream follows the pattern  $(X, X/2, X/2)$ , where  $X$  is made maximally large to satisfy the budget constraint.

Suppose that Naif has an endowment of

$$(y_1, y_2, y_3) = (3, 2, 1).$$

A second party — “Arbitrageur” — offers Naif the opportunity to trade her endowment for this alternative:

$$(c_1, c_2, c_3) = \left(2\sqrt{3/\sqrt{2}}, \sqrt{3/\sqrt{2}}, \sqrt{3/\sqrt{2}}\right) \simeq (2.91, 1.46, 1.46)$$

Naif accepts – the new sequence offers a weakly greater utility than the old one:

$$\ln \left(2\sqrt{3/\sqrt{2}}\right) + \frac{1}{2} \left[ \ln \left(\sqrt{3/\sqrt{2}}\right) + \ln \left(\sqrt{3/\sqrt{2}}\right) \right] = \ln 3 + \frac{1}{2}[\ln 2 + \ln 1].$$

Naif consumes her new claim  $c_1 = 2\sqrt{3/\sqrt{2}}$ , and period 2 begins.

Now Arbitrageur returns and offers Naif a new sequence:

$$(c_2, c_3) = \left( \sqrt[3]{2} \sqrt{3/\sqrt{2}}, \frac{\sqrt[3]{2}}{2} \sqrt{3/\sqrt{2}} \right) = (1.84, 0.92).$$

From Naif's current perspective the new sequence has a weakly higher utility than the old one:

$$\ln \left( \sqrt[3]{2} \sqrt{3/\sqrt{2}} \right) + \frac{1}{2} \ln \left( \frac{\sqrt[3]{2}}{2} \sqrt{3/\sqrt{2}} \right) = \ln \left( \sqrt{3/\sqrt{2}} \right) + \frac{1}{2} \ln \left( \sqrt{3/\sqrt{2}} \right).$$

Naif's final consumption sequence is

$$\begin{aligned} (c_1, c_2, c_3) &= \left( 2\sqrt{3/\sqrt{2}}, \sqrt[3]{2} \sqrt{3/\sqrt{2}}, \frac{\sqrt[3]{2}}{2} \sqrt{3/\sqrt{2}} \right) \\ &= (2.91, 1.84, 0.92) \\ &< (3, 2, 1). \end{aligned}$$

Her final sequence is **dominated** by her initial sequence.

- This is an example of a Dutch Book.
- A sequence of trades has strictly reduced Naif's endowment.
- This can be formally modeled as a game between a naif and a sequence of monopolist arbitrageurs who are each able to make take-it-or-leave-it offers in each period.
- The outcome that I have described is the unique (subgame perfect) equilibrium of that game.
- Aside: Dutch Book will *not* arise if the agent were sophisticated.

Aside continued: What is the equilibrium path for sophisticates?

Offer in period 1 can be constrained to take the form

$$(c_1, c_2, c_3) = (c_1, 2x, x),$$

since sophisticate anticipates that self 2 will accept an offer s.t.  $c_2/c_3 = 2$ .

Arbitrageur makes an offer in period 1 such that:

$$(c_1, x) = \arg \min c_1 + 2x + x$$

subject to

$$\ln c_1 + (1/2)[\ln(2x) + \ln x] = \ln 3 + (1/2)[\ln 2 + \ln 1].$$

Set up Lagrangian:

$$\min c_1 + 2x + x - \lambda \{ \ln c_1 + (1/2)[\ln(2x) + \ln x] - \ln 3 - (1/2)[\ln 2 + \ln 1] \} .$$

FOC:

$$1 - \lambda/c_1 = 0$$

$$3 - \lambda/x = 0$$

$$\ln c_1 + (1/2)[\ln(2x) + \ln x] - \ln 3 - (1/2)[\ln 2 + \ln 1] = 0$$

This reduces to:

$$\ln \lambda + (1/2)[\ln(2\lambda/3) + \ln(\lambda/3)] = \ln 3 + (1/2)[\ln 2 + \ln 1]$$

Unique solution is  $(c_1, c_2, c_3) = (3, 2, 1)$ .

## 2.2 Competitive markets eliminate the Dutch Book

- A competitive equilibrium is a sequence of prices and actions such that all markets clear and all agents maximize their *perceived* interests, given their beliefs.
- Theorem: Competitive markets eliminate Dutch books for time-separable, dynamically inconsistent preferences (Laibson and Yariv, 2007).

	Monopoly case	Competitive case
Initial endowment	(3, 2, 1)	(3, 2, 1)
After first trade	(2.91, 1.46, 1.46)	(3, 1.5, 1.5)
After second trade	(2.91, 1.84, 0.92)	(3, 2, 1)

- Intuition: When Arbitrageurs have to compete with one another, they offer better prices and can no longer turn individuals into money pumps.

## Possibility of Dutch Book

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	Monopoly	Competitive Equilibrium
Naif perception		
Naif outcome	✓	
Sophisticate		

### 3 Pareto-dominating trades

- Observation 2: Competitive markets do not eliminate bad outcomes.
- In competitive markets, trading might make **every** self worse off.
- In other words, even though the agent is not being Dutch booked, being allowed to freely trade might make every incarnation of the agent worse off.

- Example: Consider a Sophisticate.
- Suppose their endowment is  $(y_1, y_2, y_3) = (5/2, 2, 3/2)$ .

	Competitive case
Initial endowment	$(5/2, 2, 3/2)$
After first trade	$(3, c_2, c_3)$
After second trade	$(3, 2, 1)$

- In **competitive equilibrium**, final consumption will be

$$(c_1, c_2, c_3) = (3, 2, 1).$$

Quick proof of competitive equilibrium allocation:

- It is easy to see that in equilibrium  $c_2/c_3 = 2$ , so the problem of self 1 boils down to

$$\max \left[ \ln(6 - 3x) + \frac{1}{2} \ln(2x) + \frac{1}{2} \ln(x) \right].$$

- FOC:

$$-\frac{3}{6 - 3x} + \frac{1}{2x} + \frac{1}{2x} = 0$$

$$x = 1$$

- So the equilibrium outcome is: (3, 2, 1).

- The competitive equilibrium outcome makes **every self worse off**.
- The equilibrium outcome is Pareto-Dominated by the original endowment.
- Self 1 does not like the greater imbalance between  $c_1$  and  $c_3$ .
- Self 2 does not like the fact that she can look forward to less  $c_3$ .
- Self 3 does not like the fact that her consumption has been cut.

Could a competitive market fix the problem of Pareto Dominance?

- Yes, if Self 1 buys an optimal commitment sequence.
- It would need to be complex (to account for unmodeled stochastic contingencies)
- It would only happen if Self 1 were sophisticated (not naive).
- It would only work if the contract were not unraveled by a third party.
- It would only work if the likelihood of making a mistake were small. Irreversible contracts are risky.

What if the previous example was done with Naifs rather than Sophisticates?

Then the outcome is the same (because of  $\ln$  utility):

	Competitive case
Initial endowment	$(5/2, 2, 3/2)$
After first trade	$(3, c_2, c_3)$
After second trade	$(3, 2, 1)$

For Naifs, the free market raises the perceived welfare of self 1:

$$\ln(3) + \frac{1}{2} \ln(3/2) + \frac{1}{2} \ln(3/2) > \ln(5/2) + \frac{1}{2} \ln(2) + \frac{1}{2} \ln(3/2)$$

What is the perceived welfare improvement of self 1?

$$\frac{3}{2} \ln(3) - \ln(5)$$

What is the perceived/actual welfare loss of self 2?

$$-\frac{1}{2} \ln(3) + \frac{1}{2} \ln(2).$$

What is the perceived/actual welfare loss of self 3?

$$-\ln(3) + \ln(2).$$

Adding up the levels of perceived well-being for each self:

$$\begin{aligned} \frac{3}{2} \ln(3) - \ln(5) - \frac{1}{2} \ln(3) + \frac{1}{2} \ln(2) - \ln(3) + \ln(2) &= -\ln(5) + \frac{3}{2} \ln(2) \\ &= -\ln(2 \cdot 5/2) + \frac{3}{2} \ln(2) \\ &< \frac{1}{2} \ln(5/2) - \ln(5/2) \\ &< 0 \end{aligned}$$

In this example, “total” perceived well-being falls as a result of competitive market trading.

## Possibility of Dutch Book

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	Monopoly	Competitive Equilibrium
Naif perception		
Naif outcome	✓	
Sophisticate		

## Possibility of Pareto-Dominated Equilibrium Path

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	Monopoly	Competitive Equilibrium
Naif perception		
Naif outcome	✓	✓
Sophisticate	✓	✓

## 4 Retirement savings in 'GE'

Fadlon and Laibson (2018)

- Continuous time: working life  $[0, T)$ , and retirement  $[T, \infty)$ .
- Labor Income:  $y_i$  during working life, 0 during retirement
- Gross interest rate  $R$

- Flow utility with idiosyncratic taste shifter,  $\theta_i$ , in retirement:

$$\text{Instantaneous flow utility} = \begin{cases} u(c_{it}) & \text{if } t < T \\ \theta_i u(c_{it}) & \text{if } t \geq T \end{cases} .$$

where  $\theta_i$  is mean one, independently drawn from the cumulative distribution function  $F(\theta_i)$ , and is known to the agent at time 0.

$$u(c) = \frac{c^{1-\gamma} - 1}{1-\gamma}.$$

- Exponential discounting with discount factor  $\delta$ , with  $\delta R = 1$ .

First order condition (working life consumption  $c_{iw}$ ; retired consumption  $c_{ir}$ ):

$$\begin{aligned}u'(c_{iw}) &= \theta_i u'(c_{ir}) \\c_{iw}^{-\gamma} &= \theta_i c_{ir}^{-\gamma}\end{aligned}$$

Taking expectations

$$E \left( \frac{c_{ir}}{c_{iw}} \right)^\gamma = 1$$

If an econometrician observed  $E \left( \frac{c_{ir}}{c_{iw}} \right)^\gamma = 1$ , what would she conclude?

Classical econometrician concludes:

- Rational actor model is right.

What might a behavioral economist say?

- Three types of agents with respective masses:  $\mu_R + \mu_P + \mu_M = 1$ .
  - Rational ( $R$ ): life-time utility maximizers.
  - Passive ( $P$ ): stick to the default saving rate at their workplace.
  - Myopes ( $M$ ): consume as much as they can every period.
- Utilitarian government taxes workers at uniform rate  $\tau$ , and returns this as an annuity at retirement.
- Firms set a supplemental, optimal default savings rate  $d$ .

The *same* first-order condition that holds (on average) in the rational actor world with no government *also* holds (on average) in the behavioral world with an activist government:

$$E \left( \frac{c_{ir}}{c_{iw}} \right)^\gamma = 1.$$

This is true regardless of the proportions of myopes, passives, and rationals.

You can tell the rational and behavioral worlds apart. In the behavioral world:

1. At least mass  $\mu_M$  retire with no liquid wealth (myopes + constrained rationals).
2. Mass  $\mu_P$  use whatever default savings rate their employer sets.
3. No more than mass  $\mu_R$  are off these boundaries (rationals, except those that are liquidity constrained).
4. Institutions (like defaults and SS) are designed by social planners to address the underlying behavioral biases.

## Conclusions:

- In general equilibrium (with a paternalistic government) the economy tends to look rational (on some but not all dimensions).
- For example, in the economy that we just studied there is no average consumption 'drop' at retirement.

$$E \left( \frac{c_{ir}}{c_{iw}} \right)^\gamma = 1.$$

- *Pseudo-rationality*: rational (or quasi-rational) resource allocation achieved through paternalistic policies – not through the actions of uniformly rational households.

## 5 The Curse of Education

- Firms do not have an incentive to educate or debias consumers if debiased consumers are not profitable.
- “Curse of education”: educating the consumer makes her unprofitable.

Examples of education that will make a consumer less profitable:

- “Financial markets are nearly efficient.”
- “Bottled water is no better than tap water (in the US).”
- “The typical bank account holder pays \$90 per year in add-on fees.”

## The Curse of Education: Three Types of Profit-Lowering Education

### 1. commodification effect (competition is enhanced)

- financial markets, water

### 2. devaluation effect (willingness to pay is reduced)

- financial markets, water

### 3. cost salience effect (invisible costs are unshrouded)

- banks

## 6 Noise

- Luce-McFadden framework
- Consumers observe goods  $i$ , with true utility  $u_i$  and true price  $p_i$ :

$$(u_i - p_i) + \varepsilon_i,$$

where  $\varepsilon \sim \sigma f(\varepsilon)$ .

- Consumers pick good with highest perceived value

$$i = \arg \max_i \{(u_i - p_i) + \varepsilon_i\}$$

- **Proposition** (Perloff and Salop 1985): For identical firms

$$p - c = \frac{\sigma}{n(n-1) \int f(\varepsilon_i)^2 F(\varepsilon_i)^{n-2} d\varepsilon_i}$$

- **Proposition** (Caplin and Nalebuff 1991): Equilibrium exists if  $\ln f$  concave.
- **Proposition** (Gabaix, Laibson Li, Li, Resnick, De Vries 2016): Markups characterized in closed form by tail properties of noise distribution.

$$p - c \sim \frac{\sigma}{nf\left(F^{-1}\left(1 - 1/n\right)\right)}.$$

- **Proposition** (Gabaix, Laibson Li, Li, Resnick, De Vries 2016): For  $u_i - p_i$  bounded, and  $f$  in the domain of the logit:

$$D_i \sim \frac{\exp(\beta(u_i - p_i))}{\sum_{j=1}^n \exp(\beta(u_j - p_j))}$$

## 6.1 Does competition eliminate markups quickly?

- Uniform noise (or Cournot competition):

$$p - c \sim 1/n$$

- Exponential, logit:

$$p - c \sim 1$$

- Which intuition applies in general?

- Bounded power law noise:  $f(\varepsilon) = k(1 - \varepsilon)^{\alpha-1}$ ,  $\varepsilon \in [-1, 1]$ ,  $\alpha \geq 1$

$$p - c \sim n^{-1/\alpha} \sigma$$

- Gaussian noise

$$p - c \sim \frac{1}{\sqrt{\ln n}} \sigma$$

- Exponential noise,  $f(\varepsilon) = e^{-\varepsilon+1} \mathbf{1}_{\varepsilon > -1}$ ,

$$p - c = \sigma$$

- Log normal noise:

$$p - c \sim e^{\sqrt{2 \ln n}} \sigma$$

So  $p - c$  is not sensitive to  $n$ . Competition/entry doesn't change markups.

**Mark-ups** as a function of the number of competitors,  $n$ , with Gaussian noise and with uniform noise (or Cournot).

$n$	Gaussian noise	Uniform noise
10	1.00	1.00
100	0.61	0.1
1,000	0.48	0.01
10,000	0.40	0.001
100,000	0.35	0.0001
1,000,000	0.32	0.00001

## 6.2 Conclusions about Noise:

- Can firms exploit consumer confusion? *Yes.*  $p - c$  proportional to noise.
- Will competition decrease mark-ups? *Weakly.*  $p - c \sim \frac{1}{\sqrt{\ln n}}$
- How do firms maximize profit? *Raise complexity.*
- Will greater competition force firms to reduce complexity? *No. Complexity endogenously rises.*

- The lack of equilibrium education is a commodification effect: reducing noise makes consumers less profitable.
- With lower noise, goods become more like commodities.

# 7 Shrouded attributes

Gabaix and Laibson (2006)

- Many goods have “shrouded attributes” that some people don’t anticipate when deciding on a purchase.
- Consider buying a printer
  - Some consumers only look at printer prices.
  - They don’t look up the cost of cartridges.

- Add-ons will be shrouded and will have large mark-ups.
  - Even in competitive markets.
  - Even when demand is price-elastic.
  - Even when firm could freely unshroud the add-on...

## 7.1 Shrouded attributes

Some consumers don't consider shrouded attributes when they buy a product: Gabaix and Laibson (2006)

- Bank fees, credit card fees, mortgage closing costs (Woodward 2003).
- Credit card fees and long-term interest rates (Ausubel 1990, Agarwal et al 2006, Ru and Schoar 2018).
- Mutual fund fees: Most individual investors report that they do not know the fees that they are paying (Alexander et al. 1998, Barber et al. 2002).

- Maintenance costs
- Hotel add-on charges
- Rental car gasoline usage fees
- Printer cartridges: 3% of printer buyers report that they knew the ink price per page when they bought their printer (Hall 2003).
- Hotels (phone fees), banking (minimum balance fees), video stores (late fees) (Ayres and Nalebuff 2003, Ellison 2005).

Buying a HP deskjet printer.

- To find one, I went to the HP web site.
- Least expensive: HP Deskjet 3747 Color Inkjet Printer \$29.99
- Its recommended usage is 500 pages per month ( $< 20$  pages/day)
- What is the total cost of buying this printer and using it for four years, printing about 500 black and white pages per month?

The costs:

- \$29.99: HP deskjet 3747 plus one set of cartridges
- \$19.99: USB 2.0 printer cable, 6 feet (2 meters)
- \$ 4.99: Shipping and handling
- \$2400.00: (500 pages/month)(48 months)(\$0.10/page)

Indication that something is awry:

- Price of the printer plus first set of cartridges: \$29.99.
- Price of each replacement set of cartridges: \$29.99.

## 7.2 Bank Illustration

- Assume naive consumers do not foresee add-ons.
- Fraction  $\alpha$  of consumers are myopic.
- Fraction  $1 - \alpha$  are sophisticated.

- Basic bank account costs  $c$  for US Trust to provide.
- Firms have no market power.
- Add-on services cost 0 for firm to provide.
- Add-on services can be priced to generate fees of  $\bar{p}$  from naive consumers.
- Add-on services are avoided by sophisticates with effort cost  $e$ , where

$$e < \alpha \bar{p}.$$

Equilibrium:

- Price for opening an account:  $p$ .
- Add-on services are priced to generate fees of  $\bar{p}$ .
- Myopes pay:

$$p + \bar{p}.$$

- Sophisticates pay firm only:

$$p.$$

- Firms break even (0-profit condition):

$$p - c + \alpha\bar{p} = 0$$

$$p = c - \alpha\bar{p}.$$

- Sophisticates get a cross-subsidy:  $\alpha\bar{p}$ . Note that  $\alpha\bar{p} > e$ .
- Sophisticates would rather stick with firm that is catering to Myopes, than switch to a firm that caters to Sophisticates.

Is consumer education profitable?

What would happen if a competitor with **no** markups tried to educate consumers?

Call the new firm: Transparency Bank

- “US Trust is charging you  $\bar{p}$  for add-ons.”
- “Transparency Bank charges nothing for add-ons and a  $c$  annual fee for holding an account with us.”

Would newly minted sophisticates choose Transparency Bank?

- Sophisticates won't switch. Better to stay at US Trust, get the loss leader pricing, and avoid the add-ons.

- Net payments at US Trust for a naive:  $p + \bar{p}$ .

- Costs at US Trust for a sophisticate:

$$p + e = [c - \alpha\bar{p}] + e = c + e - \alpha\bar{p} < c$$

- Payments at Transparency for a naive:  $c$ .

- Payments at Transparency for a sophisticate:  $c$ .

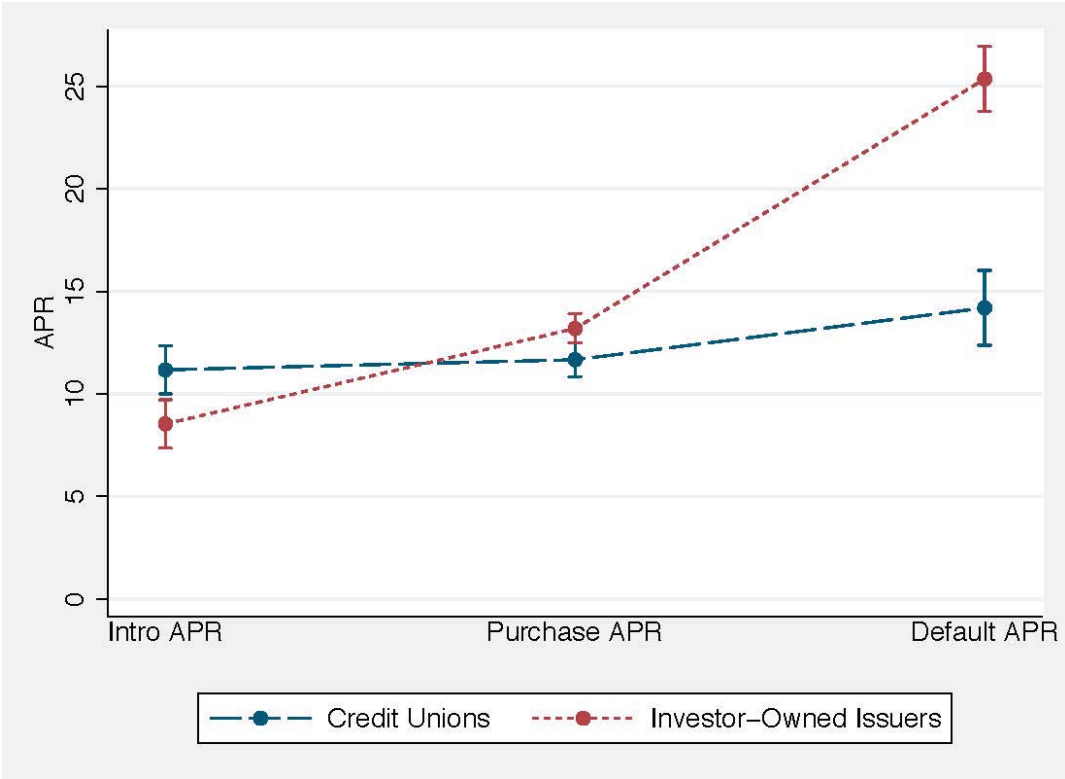
- Making consumers educated does not attract consumers to Transparency Bank (and hurts US Trust, since more consumers become sophisticated).

- Sophisticated consumers would rather pool with myopic consumers at **high** mark-up firms, then defect to low mark-up firms.
- At high mark-up firms, sophisticated consumers reap all of the benefits of loss leader base goods and avoid some of the costs of high mark-up add-ons.
  - “It’s good to stay at a hotel with an expensive spa, as long as you don’t use it.”
  - Sophisticates benefit from “free gifts” and avoid high fees.
- So advertising would make consumers sophisticated, but may **not** attract them to low mark-up firms.

## 7.3 Summary for Shrouded Attributes

- Firms set monopoly prices for add-ons.
- Add-ons are profits centers.
- Base product may be a loss leader.
- Firms shroud add-on prices.
- Firms do not educate consumers

- Making costs salient makes consumers unprofitable, even for the new entrant.
- Curse of education: educated consumers prefer to buy from firms with high markups to take advantage of cross-subsidies.



Ryan and Bubb (2010)

## Hossain and Morgan (2003)

- Researchers ran 80 auctions on eBay
- Use randomized (shrouded) shipping & handling charges for different auctions.

Shipping cost:	Revenue (including shipping cost)
\$0	\$7.54
\$3.99	\$10.14

Brown, Hossain, and Morgan (2010)

- Ran 76 iPod auctions.
- Used results to estimate the following regression, where “Disclosed” implies that the shipping fees are revealed at the top of the auction page (rather than on the check-out page).

$$\text{Revenue} = (1.13) \text{ Shipping} + (6.99) \text{ Disclosed} + (-0.47) (\text{Disclosed} \times \text{Shipping}) + \dots$$

If shipping fees are shrouded, then raising them by \$1 raises revenue by \$1.13.  
If shipping fees are disclosed, then raising them by a dollar raises revenue by  $1.13 - 0.47 = \$0.66$ .

## Ru and Schoar (2018)

- Study supply side of the credit card market (offers received through the mail)
- Card issuers target less educated customers with fees that are more backward loaded (lower introductory APR but much higher late- and over-limit fees).
- Using shocks to the credit worthiness of customers via increases in state level unemployment insurance, show that card issuers rely more heavily on backward loaded and hidden fees when customers are less exposed to negative cash flow shocks.

## 8 Conclusion

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4. Retirement Savings and Pseudo-Rationality
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